Voting not to grow: the effects of polarisation on public investment and growth

Por:
Enrique Sanz Posse[1]

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Abstract

This paper presents a political economy model of fiscal policy to study the effects of political polarisation on growth. Polarisation affects growth through the fiscal policy channel due to its effect on the composition of the government's expenditure. There are two kinds of public expenditure goods: productivity-enhancing public investment and welfare-improving district-specific goods. The first kind generates productivity growth, whereas the second kind generates present utility but does not improve productivity levels. Polarisation increases the demand for unproductive public goods at the expense of public investment because agents do not internalise the economic benefits that growth has on districts other than their own. Higher levels of polarisation thus reduce public investment, which leads to lower growth. Therefore, the process of legislative bargaining on the composition of the budget leads to an over-provision of public goods, low public investment, and lower growth when polarisation is higher.

Resumen

Este artículo presenta un modelo de economía política de la política fiscal para estudiar los efectos de la polarización política sobre el crecimiento. La polarización afecta el crecimiento a través del canal de la política fiscal, debido a su impacto en la composición del gasto público del gobierno. Existen dos tipos de bienes de gasto público: inversión pública que mejora la productividad y bienes específicos de distrito que mejoran el bienestar. El primer tipo genera crecimiento de la productividad, mientras que el segundo proporciona utilidad presente pero no mejora los niveles de productividad. La polarización incrementa la demanda de bienes públicos no productivos a costa de la inversión pública porque los agentes no internalizan los beneficios económicos que

1 Economic Analyst at Fedesarrollo's Directorate of Macroeconomic Analysis (DAMS). Master's in Economics Graduate Program (PEG) from Universidad de los Andes. Special thanks to David Pérez-Reyna and Oskar Nupia from the Universidad de los Andes, and Jenniffer Solórzano from the Bank of England. I am also grateful to Jorge Guerra, Matheo Arellano, and the Macro Research Workshop (Semillero Macro Uniandes). Thanks as well to Mauricio Cárdenas for encouraging me to study the intersection between political economy and macroeconomics. Contact: e.sanz@uniandes.edu.co

el crecimiento tiene en distritos distintos al propio. Por lo tanto, niveles más altos de polarización reducen la inversión pública, lo que conduce a un menor crecimiento. Así, el proceso de negociación legislativa sobre la composición del presupuesto lleva a una sobreoferta de bienes públicos, baja inversión pública y menor crecimiento cuando la polarización es más alta.

I. Introduction

Over the past two decades, countries across different world regions have become more politically polarised while average growth rates have slowed down. At the same time, governments have reallocated spending away from public investment and toward current expenditure. Empirical studies highlight how political economy pressures have led to an under-provision of public investment (Ardanaz & Izquierdo 2022) and an over-provision of "voter-friendly" government spending (Drazen & Eslava 2010). This paper argues that political polarisation can explain this shift in the composition of fiscal policy away from public investment and, through it, the decline in long-run growth.

I develop a dynamic model of fiscal policy under legislative bargaining to formalise this mechanism. In the model, polarisation reduces legislators' willingness to internalise the benefits of government spending provided to other electoral districts. As a result, polarised societies allocate a smaller share of spending to public investment, which drives productivity growth, and a larger share to district-specific "pork barrel" goods. The model combines the legislative bargaining framework of Battaglini and Coate (2008) with the endogenous growth under political constraints channel of Barseghyan and Battaglini (2016), to explore how political fragmentation affects the composition of fiscal policy and long-run growth. The paper thus contributes to linking the political economy of fiscal policy literature strands to long-run endogenous growth literature.

Political polarisation, as defined by the Varieties of Democracy Institute (2024), occurs when political differences permeate social relationships beyond politics itself. Societies are highly polarised if supporters of opposing camps are reluctant to engage in friendly interactions across family, civic, or workplace settings. This weakening of inter-district and inter-group ties implies that voters, and by extension their representatives in the legislature, place lower weight on the welfare of those outside their political camp or constituency. In the model, the strength of the positive spillover effect generated by district-specific "pork-barrel" captures the weaker collective incentives to finance public investment that benefits society as a whole that emerge as polarisation rises.

The paper contributes to two main strands of the literature by combining models of fiscal policy determination with those of endogenous growth. First, it extends the endogenous growth framework of Barro (1990) by embedding public investment, a productive input in firms' production functions, into a political bargaining environment. In this setting, distortionary taxation creates the usual *crowding-out* effect (Acemoglu, Golosov & Tsyvinski, 2011), but the allocation of resources between the government's current and capital expenditure now depends endogenously on political polarisation. Second, the paper builds on political economy models of fiscal policy, such as Alesina and Tabellini (1990), Tabellini and Alesina (1990), and Weingasrt, Shepsle, and Johnsen (1981), and particularly on the legislative bargaining models of Battaglini and Coate (2008) and Barseghyan and Battaglini (2016). It contributes to this literature by endogenising growth, incorporating private investment responses to fiscal policy, and evaluating the aggregate welfare effects of different voting rules.

Empirical evidence supports the notion that political economy considerations affect the composition of the budget more than its absolute size. Across emerging economies, the share of public budgets devoted to capital expenditure has declined (Izquierdo, Puig, Riera-Crichton & Vuletin, 2018). During periods of fiscal consolidation, public investment is often sacrificed in favour of current spending due to political pressures (Ardanaz & Izquierdo 2022). Similarly, the IMF (2024a) documents how political manifestos in advanced economies have reduced their emphasis on infrastructure projects relative to social spending since the 2010s. Vergne (2009) finds that politicians in developing countries shift pre-election spending toward current expenditures, while The Economist

(2024) summarises the long-run crowding-out of infrastructure investment by entitlement spending. These empirical regularities motivate the model's focus on the political determinants of expenditure composition rather than overall fiscal size.

The remainder of the paper proceeds as follows. Section 2 introduces the theoretical model and its core assumptions. Section 3 derives the equilibrium and main comparative statics. Section 4 analyses the main results, followed by section 5, which discusses the model's limitations, and presents stylised facts that highlight the relationship between polarisation and growth. Section 6 concludes.

II. Model

This section develops a dynamic political economy model that connects political polarisation with the composition of fiscal policy and long-run growth through the mechanism of legislative bargaining. The economy consists of a continuum of identical households that live infinitely and are distributed across electoral districts indexed by $i \in \{1,\ldots,n\}$. From each district, one legislator is selected at random to represent its preferences in the legislature. Since each district comprises a continuum of identical households, its representative legislator can be modelled as a representative household from district who maximises the district's utility by voting on the composition of the public budget. Fiscal policy outcomes are determined collectively through legislative bargaining. Each legislator thus acts on behalf of their district, internalising the welfare of their own district but only partially internalising the welfare enjoyed by other electoral districts.

Political polarisation is modelled as a reduction in the degree of cross-district spillovers in welfare, represented by the parameter $\chi \in [0,1]$. When $\chi=1$, districts fully internalise the welfare generated by public spending in all districts. As χ decreases, legislators assign a lower weight to the welfare of other districts. Thus, polarisation enters the model as a property of household preferences that shapes legislative bargaining behaviour through the structure of welfare spillovers.

Each household supplies labour inelastically and consumes a private good produced by a representative firm. Government expenditure consists of two components: district-specific public goods $\mathcal{G}_i$ (interpreted as current expenditure or pork barrel) and a public investment good $\mathcal{I}_t$ that augments next period's labour-augmenting technology z_{t+1} used by the representative firm. Public investment enhances productivity in the next period, thereby sustaining endogenous growth. Legislators bargain over the allocation of total spending between $\{\mathcal{G}_i\}_1^n$ and $\mathcal{I}_t$, subject to the government budget constraint financed by distortionary taxation.

Before closing, I outline the simplifying assumptions that shape the interpretation of results. First, public investment is treated as a pure public good benefiting all districts equally, while district-specific goods (pork barrel) do not contribute to productivity growth[2]. Second, agents are homogeneous and share complete information, following the new political economy tradition of Alesina and Tabellini (1990). These assumptions imply that political conflict arises from self-interest rather than asymmetric information, and that a single representative household value function summarises aggregate behaviour.

The remainder of this section proceeds as follows. Subsection A describes the model's set-up, including the household's optimisation problem, the representative firm and the government's budget constraint. Section B characterises the competitive equilibrium given a fixed fiscal policy.

2 In reality, some locally provided public goods, such as schools or transport infrastructure, may enhance productivity. This abstraction isolates the fiscal trade-off at the core of the model.

A. *Model set-up*

Households optimise consumption and saving decisions, taking fiscal policy and productivity as given. Each electoral district $i \in \{1,\ldots,n\}$ is represented by a legislator drawn at random from its population. These legislators form the national legislature, which determines fiscal policy by majority rule. The legislature votes on a fiscal package consisting of the tax rate τ, aggregate public investment $\mathcal{I}$, and the set of district-specific public goods $\{\mathcal{G}_i\}_1^n$. Following the legislative bargaining frameworks of Baron and Ferejohn (1989) and Battaglini and Coate (2007), a finite number of negotiation rounds take place each period under a closed-rule system. This setup ensures that the legislative game converges to a subgame-perfect equilibrium where the first proposal is immediately accepted.

The model features a simultaneous-move game between market participants and the legislature. Private agents optimise their decisions, taking fiscal policy as given, while the legislature chooses fiscal policy without internalising the agents' reaction functions. The solution proceeds as follows. First, I define the agents' optimisation problems and the government's budget constraint. Second, I characterise the competitive market equilibrium given fiscal policy and productivity. Third, I derive the agents' indirect utility functions, since legislators are also consumers, preferences exhibit the behavioural symmetry emphasised in the "Public Choice" tradition. Fourth, I describe the legislative bargaining process and derive the political equilibrium as a symmetric Markov-perfect equilibrium (SME). Fifth, I characterise the balanced growth path consistent with both market and political equilibria. Finally, I present welfare and comparative statics results that illustrate how political polarisation affects fiscal outcomes.

Fiscal Policy and budget constraint

Once the legislature gathers, it determines the composition of the public budget. The budget encompasses the allocation of district-specific public goods across the n electoral districts, the provision of aggregate public investment $\mathcal{I}_t$, and the tax rate τ_t. The government finances both the set of local public goods $\{\mathcal{G}_{i,t}\}_1^n$, each provided to a distinct district, and the economy-wide investment good $\mathcal{I}_t$, which enhances productivity in the subsequent period. Expenditures are financed through taxes on capital income levied at a rate τ_t. Accordingly, the fiscal policy vector in any period is given by $\tau_t, \mathcal{I}t, \mathcal{G}_{1,t}, \ldots, \mathcal{G}_{n,t}$, and the government budget must satisfy the following constraint in every period:

$$\mathcal{I}_t + \sum_{i=1}^{n} \mathcal{G}_{i,t} \leq \tau_t r_t \sum_{i=1}^{n} a_{i,t} \quad (1)$$

Firms

A representative firm operates under perfect competition, employing labour L_t and physical capital $\mathcal{K}_t$ to produce the final good. Production follows a labour-augmenting technology z_{t+1}, which evolves endogenously with public investment $\mathcal{I}_t$. The firm rents capital at the market interest rate r_t and pays a competitive wage w_t. Productivity growth depends positively on the ratio of public investment to current productivity, with a marginal effect parameter $\varphi \in (0,1)$.

$$z_{t+1} = \mathcal{I}_t^{\varphi} z_t^{1-\varphi} \quad (2)$$

Productivity growth $\frac{z_{t+1}}{z_t}$ depends on public investment scaled by current productivity because, as noted by Barseghyan and Battaglini (2016), "in a growing economy, the higher the existing level of productivity, the more expensive it becomes to improve it in absolute terms". Technological change is labour-augmenting, and aggregate output is produced according to $Y_t = \mathcal{K}_t^{\alpha}(z_t L_t)^{1-\alpha}$. Firms also face a depreciation rate δ on physical capital. The representative firm, therefore, solves the following optimisation problem:

$$\max_{\{\mathcal{K}_t, L_t\}} \{\pi_t = \mathcal{K}_t^{\alpha}(z_t L_t)^{1-\alpha} - (r_t + \delta)\mathcal{K}_t - w_t L_t\}$$

Because growth in this economy is endogenous and driven by productivity, it is convenient to detrend the model by normalising stock variables with respect to current productivity. This transformation facilitates the characterisation of the political equilibrium once the market equilibrium is defined. Let $k_t = \frac{\mathcal{K}_t}{z_t}$ denote productivity-adjusted capital. Output per efficiency unit is then given by $y_t = k_t^{\alpha} L_t^{1-\alpha}$, implying that aggregate output can be written as $Y_t = z_t y_t$. The corresponding inverse demand functions for the factors of production are therefore:

$$r_t = \alpha \frac{y_t}{k_t} - \delta \quad (3)$$

$$\frac{w_t}{z_t} = (1-\alpha)\frac{y_t}{L_t} \quad (4)$$

Households

Each representative household in district optimises lifetime utility by choosing private consumption $c_{i,t}$ levels, taking fiscal policy and productivity as given. In this model, households do not value leisure, hence they supply an inelastic unit of labour. Households also obtain utility from a concave and positive function of the public good received by its district $\mathcal{G}_{i,t}$ in the form of $f(\mathcal{G}_t) = \mathcal{G}_{i,t}^{\chi}\left(\sum_{j=1}^{n}\mathcal{G}_{j,t}\right)^{1-\chi}$ where measures the degree to which households internalise the welfare generated by public goods provided to other districts. When χ=1 , households care only about the provision of public goods in their own district. As χ decreases, they place greater weight on the total provision of public goods irrespective of their spatial distribution. The parameter ω captures the weight of public goods in household utility.

Households earn labour income at the competitive wage w_t , receive returns on capital a_t rented to firms at the interest rate r_t, and pay capital income taxes at the rate τ_t. Because all households are ex ante identical, decisions are symmetric in equilibrium, and the subscript *i* can be omitted for consumption and labour. The representative household from district *i* faces the following optimisation problem:

$$\max_{\{c_t, a_{t+1}\}} \sum_{t=0}^{\infty} \beta^t \left[\ln(c_t) + \omega \ln\left(\mathcal{G}_{i,t}^{\chi}\left(\sum_{j=1}^{n}\mathcal{G}_{j,t}\right)^{1-\chi}\right)\right] \quad (5)$$

subject to: $c_t + a_{t+1} = w_t l_t + (1 + (1-\tau_t)r_t)a_t$

The representative household's optimal conditions are:

Euler's equation:

$$\frac{c_{t+1}}{c_t} = \beta(1 + (1-\tau_{t+1})r_{t+1}) \quad (6)$$

Budget constraint:

$$c_t + a_{t+1} = w_t l_t + (1 + (1-\tau_t)r_t)a_t \quad (7)$$

Given fiscal policy and productivity levels, the market equilibrium determines the agents' optimal consumption and saving decisions. Using these outcomes, I derive the households' indirect utility functions, which summarise how fiscal choices map into welfare. These functions are then used to define the legislators' objectives in the political game, where they bargain over the allocation of public spending and the tax rate.

B. *Market equilibrium given a fixed fiscal policy*

The market equilibrium is symmetric since households are *ex-ante* homogeneous and thus $a_{i,t} = a_t$, $c_{i,t}$ for all *i* districts. Households and firms operate, taking the government's fiscal policy as given, since they neither control the legislative bargaining process. Given that the representative agent from each electoral district is drawn at random from a continuum of households in each district, the probability of any single household becoming the representative and determining the shape of the fiscal policy is zero.

Definition 1 (Symmetric Market Competitive Equilibrium given a fixed fiscal policy). *A symmetric competitive market equilibrium comprises a set of allocations to each representative household* $\{c_t, l_t, a_{t+1}\}$*, productive inputs employed by the representative firm* $\{\mathcal{K}_t, L_t\}$ *, and a set of prices* $\{w_t, r_t\}$ *such that, given a fixed set of fiscal policy instruments* $\{\tau_t, \mathcal{I}_t, \mathcal{G}_{1,t}, \dots, \mathcal{G}_{n,t}\}$*, the current productivity level* Z_t*, and fixed initial conditions* $\{\mathcal{K}_0, z_0\}$ *the following conditions are met:*

1. *Given the set of prices, the government budget is balanced. That is, equation* 1 *is satisfied with equality.*

2. *Productivity growth follows equation* 2.

3. *Given the set of prices, the productive inputs are the solution to the representative firm's benefit-maximisation problem. That is, they fulfil equations 3 and* 4.

4. *Given the set of prices, households optimise their lifetime utility function. That is, the allocation of inputs satisfies equations 6 and 7.*

In addition, given the set of prices, markets clear, and the following conditions are met:

Labour:

$$n\, l_t = L_t \quad (8)$$

Capital:

$$n\, a_{t+1} = \mathcal{K}_{t+1} \quad (9)$$

Final good:

$$n\, c_t + \mathcal{K}_{t+1} - (1-\delta)\mathcal{K}_t + \mathcal{I}_t + \sum_{i=1}^{n} \mathcal{G}_{i,t} = Y_t \quad (10)$$

Let private investment net of depreciation be denoted by $\mathcal{S}_{t+1} = \mathcal{K}_{t+1} - (1-\delta)\mathcal{K}_t$. The following equation expresses the share of total output devoted to private consumption, denoted by a capital C_t:

$$C_t(\mathcal{S}_{t+1}, \mathcal{I}_t, \mathcal{G}_t) = \frac{1}{n}\left(Y_t - \mathcal{S}_{t+1} - \mathcal{I}_t - \sum_{i=1}^{n} \mathcal{G}_{i,t}\right)$$

To solve the model along a balanced growth path, I adjust the endogenous variables with the existing levels of productivity. Define $k_t = z_t \mathcal{K}_t$ as the stock of capital per efficiency unit. This measure is for ease of the mathematical expression and allows for detrending the model at a balanced growth path (later defined).

C. Indirect Utility Functions

To characterise the political equilibrium, I derive the representative household's indirect utility function. It is useful to define some variables as fractions of total production in the following manner:

$$I_t = \frac{\mathcal{I}_t}{Y_t}, G_{i,t} = \frac{\mathcal{G}_{i,t}}{Y_t} \text{ and } S_{t+1} = \frac{\mathcal{S}_{t+1}}{Y_t}.$$

Let $\mathcal{H}(k_t, G_t, I_t) = k_{t+1}$ be the house hold's reaction function for savings given the existing capital stock and government spending. For ease of notation, define lower case $u(k_t, G_t, I_t) = \ln\left(\frac{C(k_t, G_t, I_t)}{n}\right)$ as a term of the utility function that depends on the share of total income devoted to consumption.

Lemma 1. *The household's utility function (Equation* 5*) can be rearranged as:*

$$\mathcal{U}_i(k_t, G_t, I_t) = u(k_t, G_t, I_t) + (1+\omega)\ln(Y_t)$$
$$+ \omega \ln\left(G_{i,t}^{\chi}\left(\sum_{j=1}^{n} G_{j,t}\right)^{1-\chi}\right)$$

Subsection A2. of the appendix contains the proof of Lemma 1.

The choice of functional form implies that the household's indirect utility function is separable into three components. The first term captures the utility derived from private consumption, which depends on the share of disposable income allocated to consump-

tion. The second term reflects the direct effect of aggregate output on welfare: for a given consumption share, higher output raises utility. The third term represents the utility obtained from district-specific public goods as a share of total output. This structure allows policymakers to anticipate how fiscal policy decisions affect welfare through both private and public channels.

Notably, the tax rate τ_t does not appear explicitly in the indirect utility representation. This follows from the balanced-budget constraint and from the fact that tax revenues not allocated to current public goods finance public investment, which in turn determines future productivity growth. Hence, once the legislature chooses the allocation between public goods and public investment, the implied tax rate adjusts endogenously to balance the budget.

Finally, the equations describing the household's behaviour in market equilibrium can be expressed as reaction functions to the set of public policies. Consumption is expressed as a constant share of total output, and primes denote next-period variables.

Productivity-adjusted output:

$$y(k, G, I) = k^{\alpha} L^{1-\alpha}$$

Productivity growth:

$$\mathcal{Z}(k, G, I) = (Iy)^{\varphi}$$

Private savings:

$$\mathcal{H}(k, G, I) = k'$$

Tax rate:

$$T(k, G, I) = \frac{y\,(I + \sum_{i=1}^{n} G_i)}{\alpha y - \delta k}$$

Consumption (Share of output):

$$C(k, G, I) = 1 - \mathcal{Z}\left(\frac{\mathcal{H}}{y}\right) + (1 - \delta)\frac{k}{y} - \left(I + \sum_{i=1}^{n} G_i\right)$$

Netprivate investment (Share of output):

$$S(k, g, I) = \frac{z'}{z}\left(\frac{\mathcal{H}}{y}\right) - (1 - \delta)\frac{k}{y}$$

Euler equation:

$$\mathcal{Z}\frac{y'}{y}\frac{C(\mathcal{H}, G'', I')}{C(k, G, I)} = \beta\left(1 + (1 - T')\left(\alpha\frac{y'}{k'} - \delta\right)\right)$$

In a balanced growth path, endogenous variables grow at the same rate as productivity growth, which in turn depends on the level of public investment. Therefore, endogenous productivity growth requires a political explanation of the formation of fiscal policy (Barseghyan & Battaglini 2016). Let $\gamma = \frac{z_{t+1}}{z_t}$ be a constant which defines the balanced growth path as follows:

Definition 2 (Balanced growth path). *A balanced growth path in this economy occurs when all endogenous variables grow at the same rate as productivity grows, and this occurs at a constant rate. Then, the economy is at a balanced growth path when* $\forall x \in \{c, \mathcal{K}, Y\}$ *it occurs that* $\frac{x_{t+1}}{x_t} = \gamma$

The relation $\frac{\mathcal{I}_t}{z_t}$ needs to be constant in a balanced growth path since productivity growth depends on the level of public investment $\mathcal{I}$. For it to be constant, public investment in nominal terms needs to grow at the same rate as productivity growth and $\gamma = (I_s y_s)^{\varphi}$. Therefore, to determine the balanced growth path in this economy, the political process which determines needs to be explained

III. Political Equilibrium

The legislature meets with n legislators, each drawn at random from every electoral district. The legislative bargain process follows the model by Battaglini and Coate (2008) and Barseghyan and Battaglini (2016). All households in each district are homogeneous; the only source of heterogeneity is the district each household belongs to. Therefore, the random draw is equivalent to selecting a representative household from each district. The legislature meets at the start of the period once the level of existing capital stock k and the current level of productivity z are known.

The formation of fiscal policy occurs as in Battaglini and Coate (2008) . The legislators take turns to become the proposer and present a fiscal package, on which the legislature votes ot approve or not. The proposer is a member of the legislature chosen at random and presents a policy package $\{\tau_t, \mathcal{I}_t, \mathcal{G}_{1,t}, \dots, \mathcal{G}_{n,t}\}$. Then, members of the legislature vote on the proposal, which requires the approval of m representatives such that $1 \leq m \leq n$. The cycle of selecting a proposer at random and then voting on the platform is repeated if no agreement is reached. Each cycle takes negligible time. After $R \geq 2$ rounds, a default policy platform is implemented if no agreement is reached. The default platform fulfils the government budget restriction and issues uniform public goods across districts ($\underline{\mathcal{G}} = \mathcal{G}_{s,t}$ for all i).

Three further assumptions are necessary to guarantee an interior solution.

First, the existence of lower limits on the provision of public goods $G_i \geq \underline{G}$ for all i. The lower limit $\underline{G} \geq 0$ implies that the proposer cannot offer a negative district-specific public good, and if $\underline{G} = 0$, then it can offer no pork barrel at all. This assumption represents the lower limit on public goods provisions (e.g. constitutionally mandated entitlements) and prevents the proposer from offering zero amount of *pork barrel* in equilibrium when $m > 1$.

Second, a minimal investment in pure public goods is necessary for government and market institutions to operate. Therefore, public investment cannot fall below a certain threshold $I \geq \underline{I}$ since $\mathcal{I}$ is a pure public good. In the case of public investment, the lower bound represents the threshold below which private exchange cannot occur due to a lack of infrastructure (either physical or immaterial, such as the judiciary). In the case of current public goods, the lower bound represents existing entitlements (e.g. pensions) or court-mandated expenditures. Lastly, $\tau \in [0,1]$ is a necessary assumption to bound the problem.

Therefore, at any given round of negotiations, the representative selected to propose a policy platform needs to obtain the approval $m - 1$ of other legislators, forming a coalition. The proposer seeks to maximise its own utility subject to guaranteeing that at least $m - 1$ other legislators approve its fiscal package.

Following the base models of Battaglini and Coate (2008), Barseghyan and Battaglini (2016), and (Klein, Krusell & Ríos-Rull, 2008), the political equilibrium I use in this model is a symmetric Markov perfect equilibrium (SME), which is a refinement of a sub-game perfect equilibrium. The strategies of the legislators depend on the levels of capital stock adjusted by productivity with $k = \frac{\mathcal{K}}{z}$. Therefore, the strategies of the proposer are conditioned only on the state variables. This equilibrium does not depend on past actions nor events, which excludes reputation and signalling as potential strategies (Occhino 2012; Klein et al. 2008). Since households are *ex-ante* homogeneous, the identities of the proposer and of the minimum winning coalition do not matter, and thus the equilibrium is symmetric. Since the decisions of a legislature affect the state variables that the next legislature will face, the equilibrium considered is of a Markov nature as a dynamic game between successive governments. Moreover, since the first proposal is immediately accepted, the strategies form a subgame perfect equilibrium of the game. Since the strategies depend only on the state variables, the game only encodes payoff-relevant in-

formation, and the strategies form a subgame perfect equilibrium, the SME is a refinement of a subgame perfect equilibrium.

I further refine the SME and determine that the only relevant state variable is the level of capital stock. The proposer's strategy space consists of a policy platform; thus, the SME consists of policy functions that depend on capital levels

$$\mathcal{P} \equiv \{\mathcal{T}(k), I(k), G(k)\}.$$

The policy function $\mathcal{T}(k)$ is the tax rate level, $I(k)$, is the function that determines the share of total product devoted to public investment. The last function, $G(k)$, is the amount of public goods which the proposer from district *i* offers to its own district. Therefore, $G(k) = G_i$ in equilibrium, as it will be shown later. Given that in equilibrium the levels of pork barrel offered to the $m - 1$ members of the winning coalition, represented by G_m depend on G_i, the subscript can be suppressed for ease of notation and $G(k) = G$. It must be noted, the policy functions are defined as fractions of output rather than their nominal amount, and since the existing level of productivity is determined, the relevant state variable is the level of productivity-adjusted capital *k*.

The political game guarantees that the proposer only needs to issue a proposal on $G(k)$ rather than one level of public goods for each of the *n* districts. This results because the proposer only needs a winning coalition and all members are *ex-ante* homogeneous. Therefore, it is as if the proposer chooses at random $m - 1$ other legislators to form a coalition and can ignore the utility of $n - m$ the members of the opposition. Therefore, it will offer to the $n - m$ non-coalition members the least possible amount of public goods $\underline{G}$ and for the $m - 1$ coalition members it will offer a level of public goods G_m which directly depends on $G(k)$ (the amount of pork barrel it wants to send to its own district). The tax rate $\mathcal{T}(k, G, I) = \tau$ because the budget needs to be balanced. Therefore, once the agenda-setter issues proposals on *G* and *I*, $\mathcal{T}$ adjusts accordingly to balance the budget. Hence, $\mathcal{P}$ only has three elements.

Legislators vote in favour of a proposal if they prefer it (albeit weakly) to continuing to the next negotiation round, in which they face uncertainty on whether they will be members of the opposition, the winning coalition or become the next round's agenda-setter. Working backwards from the final round, an equilibrium can be found in which the first proposal is approved immediately at the first round by a winning coalition. Since the elements of $\mathcal{P}$ depend on *k*, the value function can be expressed only in terms of *k* and the policy variables.

To solve the political equilibrium, I first define the value function and its continuation value. This takes as input the expected value of both the indirect utility (before the legislature gathers agents ignore whether they will be the proposer, part of the winning coalition or in opposition) and its continuation function with an expected value operator. Then, I describe the process through which the legislature bargains and approves a policy platform. Agents vote in favour of a proposal if they prefer it to passing on to the next negotiation round. Afterwards, I present the problem faced by the proposer, or agenda setter, and lastly, I characterise the SME. **Subsection A details the legislative bargaining process. Subsection B characterises the problem faced by the legislator acting as the proposer. Subsection C defines the symmetric Markov equilibrium and the steady state.**

At the start of each period and before the identity of the proposer and the winning coalition members is revealed, agents face the following value function, which is the present-period utility from Lemma 1 and its continuation function, which is the present-period utility iterated one period forward.

$$V(k,z) = \max_{\mathcal{P}} \left\{ u(k,G,I) + (1+\omega)\ln(zy) + \omega \mathbb{E}\left[\ln\left(G_i^{\chi} \left(\sum_{j=1}^{n} G_j \right)^{1-\chi} \right) \right] + \beta \mathbb{E}[V(k',z')] \right\}$$

The proposer faces the following problem:

$$V^{p}(k,z) = \max_{\mathcal{P}} \left\{ u(k,G,I) + (1+\omega)\ln(zy) + \omega \ln\left(G_i^{\chi} \left(\sum_{j=1}^{n} G_j \right)^{1-\chi} \right) + \beta \mathbb{E}[V(k',z')] \right\}$$

$$\text{subject to } y\left(I + \sum_{i=1}^{n} G_i \right) = \tau(\alpha y - \delta k)$$

Also, $G_i \geq \underline{G}$ and $I \geq \underline{I}$ and $u(k,G,I) = ln(C(k,G(k),I(k))) - ln(n)$.

A. Legislative bargaining

Since negotiations take place and there is an upper limit $R \geq 2$ on the number of rounds, incentives exist for a proposal to be accepted at any point to prevent the default distribution of public goods.

1. At the last round, the proposer has incentives to marginally increase the provision of public goods to a winning coalition of $m-1$ districts and its own district at the expense of the $n-m$ members of the opposition.

2. This guarantees that a proposal is accepted at most at the last round.

3. Iterating backwards, the same incentives exist in each round. Since the proposer in any one round cannot be certain of belonging to the winning coalition of future rounds, the incentive exists for the proposer to maximise the chances of its proposal being accepted.

4. Further iterating backwards, the proposer of the first round has all the incentives to see its proposal accepted immediately. This is the result.

In each round, three different levels of public good are proposed. For the proposer's district G, for the districts of the $m-1$ members of the coalition, the proposal is G_m, and for the rest they receive the lowest possible amount $\underline{G}$. That is, with a $1/n$ chance, the agent will be the proposer and thus be able to receive G pork barrel. With a $\frac{m-1}{n}$ chance, it will be part of the winning coalition and receive G_m of pork barrel. Lastly, with a $\frac{n-m}{n}$ chance, the agent will be part of the opposition and receive the minimal amount of public goods transfers. At the start of each round, the expected value of receiving a given level of public good is

$$\mathbb{E}\left[\ln(G_i)\right]|_{i\in\{1,\dots,n\}} = \frac{1}{n}\left(\ln(G) + (m-1)\ln(G_m) + (n-m)\ln(\underline{G})\right)$$

Let $V_c(k,z)$ be the expected value function of any given representative at the start of the negotiation round. This becomes the outside option of a member of the coalition voting against the proposal and risking moving the election to the next round of negotiations, where it cannot be certain of being included again in the winning coalition:

$$V_c(k,z) = u(k) + (1+\omega)\ln(zy) + \omega \mathbb{E}\left[\ln\left(G_i^{\chi} \left(\sum_{j=1}^{n} G_j \right)^{1-\chi} \right) \right] + \beta \mathbb{E}[V(k',z')]$$

The proposer needs to guarantee that its proposal improves the expected utility of the m members of the coalition who receive G_m . Therefore, the *incentive compatibility* constraint is:

$$u(k) + (1+\omega)\ln(zy) + \omega\ln\left(G_m^{\chi}\left(\sum_{j=1}^{n} G_j\right)^{1-\chi}\right) + \beta\mathbb{E}[V(k',z')] \geq V_{\mathcal{C}}(k,z)$$

Proposition 1.
In equilibrium, $G_m = G^{\frac{1}{n-m+1}}\underline{G}^{1-\frac{1}{n-m+1}}$ *satisfies the incentive compatibility constraint*

Subsection A4. of the appendix contains the proof of 1.

This result implies that the level of public goods offered to the members of the winning coalition that satisfies the incentive compatibility and guarantees their vote is a geometric average of the level of public good the proposer sends to its own district and the lowest level of public good it can offer to the rest. Using the result from proposition 1, the total amount of public goods $G^T(k)$ becomes:

$$G^T(k) = G(k) + (m-1)\left(G(k)^{\frac{1}{n-m+1}}\underline{G}^{1-\frac{1}{n-m+1}}\right) + (n-m)\underline{G}$$

Also, the expected value of receiving public goods at the start of each round becomes:

$$\mathbb{E}[\ln(G_i)]|_{i\in\{1,\dots,n\}} = \frac{1}{n-m+1}\ln(G) + \left(\frac{n-m}{n-m+1}\right)\ln(\underline{G})$$

B. *Proposer's problem*

Proposition 1 implies that the total amount of public goods $G^T(k)$ is a function of the amount of public goods destined to the proposer's district $G(k)$. Additionally, $V^P(k,z)$ expresses the indirect utility function as separable between z and the rest of the variables from the proposer's problem stemming from the indirect utility function (11) as:

$$V^p(k,z) = \max_{\mathcal{P}} \{(1+\omega)\ln(z) + u(k) + (1+\omega)\ln(y) + \omega\chi\ln\big(G(k)\big) + \omega(1-\chi)\ln\big(G^T(k)\big) + \beta V(k',z')\}$$

subject to:

- $y\left(I + \sum_{i=1}^{n} G_i\right) = \tau rk$
- $G^T(k) = G(k) + (m-1)\left(G(k)^{\frac{1}{n-m+1}}\underline{G}^{1-\frac{1}{n-m+1}}\right) + (n-m)\underline{G}$
- $\frac{z'}{z} = (Iy)^{\varphi}$
- $G \geq \underline{G}, I \geq \underline{I}, \tau \in [0,1]$

The following equations characterise the solution to the proposer's problem:

Output:

$$y(k,G,I) = k^{\alpha}L^{1-\alpha}$$

Productivity growth:

$$\mathcal{Z}(k,G,I) = (Iy)^{\varphi}$$

Private investment:

$$k' = \mathcal{H}(k,G,I)$$

Total public goods:

$$G^T(k) = G(k) + (m-1)\left(G(k)^{\frac{1}{n-m+1}}\underline{G}^{1-\frac{1}{n-m+1}}\right) + (n-m)\underline{G}$$

Pork barrel:

$$G_m(k) = G(k)^{\frac{1}{n-m+1}}\underline{G}^{1-\frac{1}{n-m+1}}$$

Government budget:

$$\mathcal{T}(k,G,I) = \frac{y\left(I + G^T(k)\right)}{\alpha y - \delta k}$$

Final good:

$$C(k,G,I) = 1 - \mathcal{Z}\left(\frac{\mathcal{H}}{y}\right) + (1-\delta)\frac{k}{y} - \left(I + G^T(k)\right)$$

Euler equation:

$$\mathcal{Z}\frac{y'}{y}\frac{C(\mathcal{H},G'',I')}{C(k,G,I)} = \beta\left(1 + (1-T')\left(\alpha\frac{y'}{k'} - \delta\right)\right)$$

In order to solve the proposer's problem in an SME, I further simplify and isolate the productivity from the endogenous variables. Since future productivity is determined by the path that public investment takes, once this path is discounted, the effects of future public investment can be taken into account. Therefore, the value function that solves the SME does not include future levels of productivity since, in equilibrium, these depend endogenously on the decisions made on *I*. This will guarantee that the SME is achieved regardless of the level of productivity and will only depend on the state variable *k* (productivity-adjusted capital).

Proposition 2. $v(k,z) = \mathcal{A}\ln(z) + \mathcal{V}(k)$ *is the value function where* $\mathcal{A}$ *is a constant and* $\mathcal{V}(k)$ *is a recursive function:*

$$\mathcal{V}(k) = U(k,G,I) + \frac{\omega\chi}{n-m+1}\ln\left(G(k)\right) + \omega(1-\chi)ln(G^T(b,k)) + \beta\mathcal{V}(\mathcal{H}(k,G,I)$$

where:

- $U(k,G,I) = \left(\frac{\beta\varphi(1+\omega)}{1-\beta}\right)\ln(I) + u(k,G,I) + (1+\omega)\left(\frac{\beta(\varphi-1)+1}{1-\beta}\right)\ln(y)$;
- $u(k,G,I) = ln(C(k,G,I)) - \ln(n)$
- $C(k,G,I) = 1 - \mathcal{Z}\left(\frac{\mathcal{H}}{y}\right) + (1-\delta)\frac{k}{y} - (I + G^T(k))$.

Subsection A4.a. of the appendix contains the proof of proposition 2. Taking stock of these results, I rearrange the proposer's problem in the following fashion:

$$\mathcal{V}^p(k,z) = \max_{\mathcal{P}}\left\{\left(\frac{\beta\varphi(1+\omega)}{1-\beta}\right)\ln(I) + u\left(C(k,G,I)\right) + (1+\omega)\left(\frac{\beta(\varphi-1)+1}{1-\beta}\right)\ln(y) + \omega\chi\ln\left(G(k)\right) + \omega(1-\chi)\ln\left(G^T(k)\right) + \beta\mathcal{V}(\mathcal{H}(k,G,I)\right\}$$

$$\text{subject to}\left\{\begin{matrix}\mathcal{T}(k,G,I) = \frac{y(I+G^T(k))}{\alpha y - \delta k} \\ G \geq \underline{G}, I \geq \underline{I}, \tau\epsilon[0,1]\end{matrix}\right\}$$

where:

$$\left\{\begin{array}{c} u\big(C(k,G,I)\big) = ln(C(k,G,I)) - \ln(n) \\ C(k,G,I) = 1 - \mathcal{Z}\left(\frac{\mathcal{H}}{y}\right) + (1-\delta)\frac{k}{y} - (I + G^T(k)) \end{array}\right\}$$

Using the result from the previous proposition, I express the value function as $v = \mathcal{A}\ln(z) + \mathcal{V}(k)$, where $\mathcal{A}$ is a constant and $\mathcal{V}(k)$ does not depend on productivity levels. Then, I rearrange the proposer's problem as follows: First, substitute the dynamics of productivity growth into the government budget. Next, replace the interest rate with the market-determined interest rate. Then, incorporate total output into the value function, and finally, I include the indirect labour supply, as the government can anticipate the effects of its decisions on agents.

For simplicity, define $F(G;\chi) = \omega\chi\ln(G(k)) + \omega(1-\chi)\ln(G^T(k))$ as the utility derived from the consumption of public goods. Let primes denote forward lags and subscripts partial derivatives. When the government optimises with $k' = \mathcal{H}(k, G(k), I(k))$ as given and plays in a simultaneous game with the households, the government does not internalise the household's reaction functions to its own problem. Hence, the proposer's optimality conditions become:

Intratemporal: Demand for *I*

$$\frac{A_0}{I} = u_c\left(1 + \frac{\mathcal{Z}_I}{y}\mathcal{H}\right)$$

Intertemporal *G* vs *I*:

$$\frac{\partial \mathrm{F}}{\partial G} = \frac{A_0}{I} - u_c\left(1 - \frac{\partial G^T}{\partial G} + \frac{\mathcal{Z}_I}{y}\mathcal{H}\right)$$

Tax-rate τ=T:

$$\mathrm{T}\big(k, G(k), I(k)\big) = y\left(\frac{I(k) + G^T(k)}{\alpha y - \delta k}\right)$$

C. Symmetric Markov Equilibrium

In a simultaneous game with households, market exchange occurs concurrently with the legislative bargaining process. Now that I rearranged and detrended the proposer's problem, leaving it with only one endogenous state variable *k*, I describe the political equilibrium following the characterisation of Markov perfect Equilibria of Klein et al. (2008) and Occhino (2012). The legislature thus decides on the share of total output devoted to pork barrel and public investment. Hence, in equilibrium, a steady state exists along a balanced growth path. Define both capital and output as productivity-adjusted variables (their nominal value is simply multiplied by the level of productivity). I present some comparative statics on the values of a steady state for a given level of productivity.

Definition 3 (Symmetric Markov Perfect Equilibrium). *A Symmetric Markov Perfect Equilibrium (SME) is a set of a value function v(k,z), the set of the government's associated policy functions $\mathcal{P} \equiv$ {T(k),I(k),G(k)}, the private sector's associated policy functions {C(k,G,I),H(k,G,I)} and prices {r,w} such that given an existing level of capital k and productivity z at the start of each period:*

i. The market equilibrium (Definition 1) and {C(k,G,I), H(k,G,I)} are optimal reaction functions given $\mathcal{P}$.

ii. The value function is v(k,z)=$\mathcal{A}$ln(z)+V(k) where $\mathcal{A}$ is a constant V(k) and is defined recursively.

iii. The proposer maximises V^P (k,G,I) and $\mathcal{P}$ is optimal given given {C(k,G,I),H(k,G,I)}.

iv. Given $\mathcal{P} \equiv$ {T(k),I(k),G(k)} and {C(k,G,I),H(k,G,I)} and:

$$\mathcal{V}^p(k,z) = \left\{ \begin{array}{l} \left(\frac{\beta\varphi(1+\omega)}{1-\beta}\right)\ln(I) + u(C(k,G,I)) + (1+\omega)\left(\frac{\beta(\varphi-1)+1}{1-\beta}\right)\ln(y) \\ +\omega\chi\ln(G(k)) + \omega(1-\chi)\ln(G^T(k)) + \beta\mathcal{V}(\mathcal{H}(k,G,I)) \end{array} \right\}$$

IV. Main results

Polarisation leads to a higher provision of pork barrel and lower public investment at steady-state values determined by the Symmetric Markov Equilibrium (Definition 3). First, polarisation increases the demand for pork barrel by the proposer's district and by the members of the coalition. Higher polarisation thus increases the steady-state level of public goods through the proposer's optimality condition for *G*. Second, greater levels of polarisation increase the utility of one unit worth of public goods directed to the proposer's district when compared with the utility which public investment can produce. Hence, it is through the demand for pork barrel that higher public spending generates distortions in this economy.

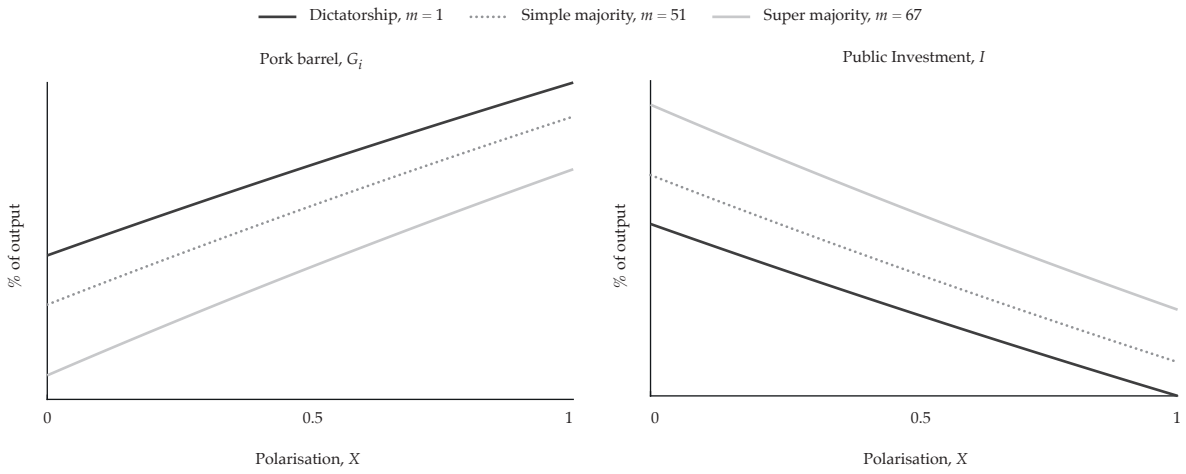
The effects of polarisation on growth filter through its relation with the three externalities I discussed earlier. The first is its inverse relation with the positive spillover effects of pork barrel. Higher polarisation weakens the spillover effect and thus agents value more the provision of pork barrel G_i to their own district rather than the provision of public goods to all districts. The second channel is through its negative effect on the provision of public investment. Since both pork barrel and investment are financed through common resources (tax revenue), when polarisation is high, agents perceive a higher opportunity cost of using taxes to finance public investment when they could be spent on pork barrel. Therefore, because government spending will be targeted only towards members of the winning coalition, the incentives to generate growth become weaker. Because the proposer cannot be sure of its district being part of the winning coalition in future periods, every unit spent on public investment has an opportunity cost of being used to finance pork barrel or reduce taxes, since its externality affects the future and not the present. Therefore, the positive externality of public investment is also connected with he negative externality of levying taxes.

To better understand the main results from this model, I present some graphical cualitative exercises of comparative statics on the level of polarisation χ. The following graphs serve an illustrative purpose to compare different variables under a balanced growth path. Additionally, I present results using different values of m to study the effects of voting rules. The first exercise (comparative statics on χ allows us to study the main results of this model. The second one allows us to draw some normative and policy implications of the model, in which voting rules constitute part of the institutional framework.

The results are presented under different voting rules. Comparative statics on the voting rules required by the legislature to approve a fiscal policy platform, here captured by the parameter m, are relevant as a normative implication of this model. When $m = 1$, only one vote is required, and thus this scenario constitutes a dictatorship, where only the will of the proposer is taken into account. When $m = 51$, a simple majority is required, and when $m = 67$, a qualified majority is needed to approve a fiscal policy platform. The comparison between the two democratic voting systems helps to understand the costs of collective action. When $m = 100$, voting rules require a unanimous decision, which is interpreted as a world without political frictions. In this model, since I normalised $n = 100$, m can represent a percentage of the total votes required.

Polarisation ultimately leads to lower consumption and lower growth levels. The transmission from polarisation to lower growth works through its effects on fiscal policy. The definition of polarisation I chose implies that as polarisation rises, agents interact less with members of external electoral districts. This weakens the positive spillover effects as agents will value the provision of public goods to districts other than their own less. Therefore, polarisation first increases the demand for pork barrel G for the proposer's district and the coalition at the expense of lower public investment. Since I induces economic growth which benefits all districts, greater polarisation increases the opportunity cost of public investment compared with pork barrel, which directly increases the utility of the proposer's district and the winning coalition (see figure 1).

Figure 1:
Pork barrel G_i for the proposer's district vs Public Investment I



However, because the incentive compatibility constraint implies that the fall in demand for public investment is higher than the increase in the demand for pork barrel, and thus total government spending falls as polarisation rises. When the incentive compatibility constraint is fulfilled, the amount of pork barrel that the proposer offers to the coalition members becomes a geometric average between the pork barrel it sends to its own district (G_i) and the lower bound $\underline{G}$. Therefore, every unit of output devoted to public investment (which benefits all districts equally) generates a higher marginal utility for the proposer to consume as pork barrel, and thus, in equilibrium, for the same amount of lifetime utility, the proposer needs to devote a lower share of output to pork barrel for its own district and its coalition. The result implies that total government expenditure is decreasing as polarisation rises along a balanced path with a given steady-state productivity z (see figure 2).

Additionally, relative to a dictatorship, government spending is higher under lower levels of polarisation and lower under higher levels of polarisation. The threshold occurs because under a dictatorship, public investment is lower and pork barrel is higher. Therefore, when polarisation rises under a dictatorship, the demand for pork barrel increases relative to the fall in public investment at a faster rate than it does under more democratic voting rules.

Figure 2:
Total government spending G^T

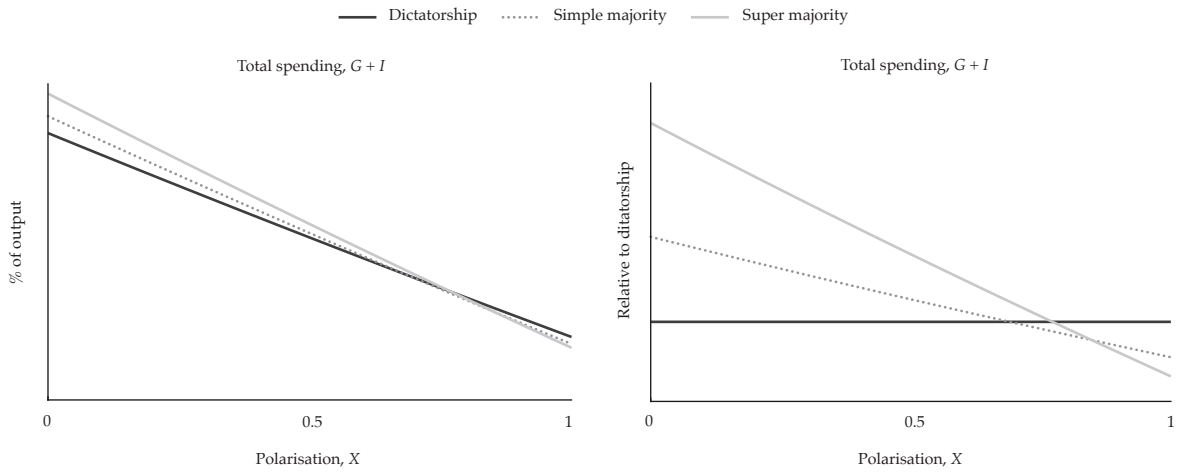
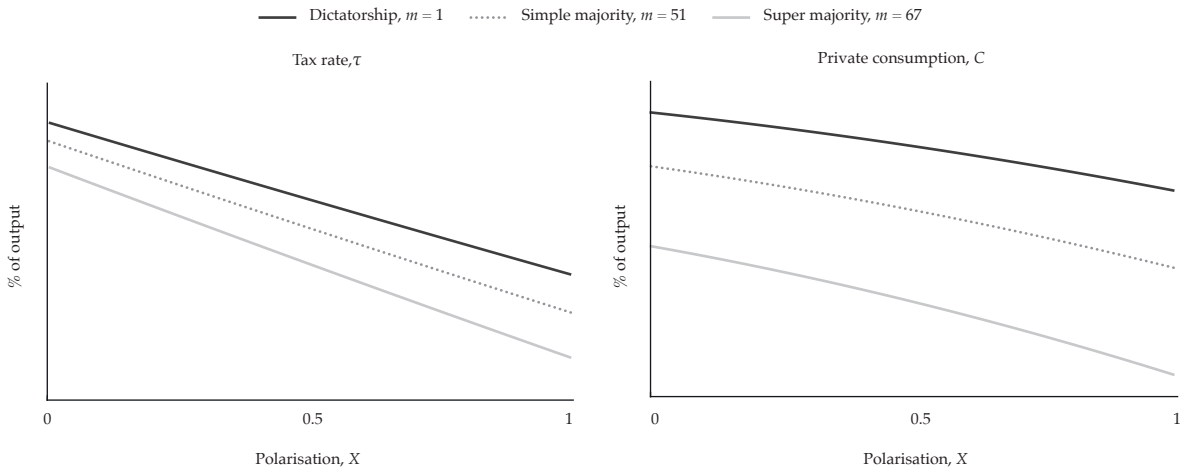


Figure 3:
Tax rates τ and private consumption C



Second, as the demand for public investment falls, and the members of the opposition do not receive public current spending, tax rates τ fall. The proposer internalises the possibility of not being in office for future periods, and therefore, it does not control the path of future fiscal policies. Because when polarisation increases, the opportunity cost of raising taxes to finance public investment increases. Proposers seek to improve the utility of their own district, and hence the lower tax rate, because one additional unit of G has a higher marginal value than an additional unit of I. Lower demand for I facilitates a lower tax rate. Additionally, because pork barrel increases and the tax rates decrease, households save more and thus consume C less. Figure 3 shows these results on the policy functions.

Third, lower tax rates encourage private investment as polarisation increases, which leads to a lower interest rate. Due to higher private savings and a higher accumulation of private capital S, steady-state levels of capital and output per efficiency unit, k and respectively, are increasing in polarisation. This result implies that wage rates w are also increasing in polarisation as a direct result of greater private capital accumulation. Figure 4 shows these results on interest and wage rates. Fourth, lower tax rates incentives higher private savings S by increasing the after-tax rate of return. This increase in private savings thus generates an increase in the balanced-growth level of capital per efficiency unit k (See figure 5).

Figure 4:
Interest rates r and wage rate w

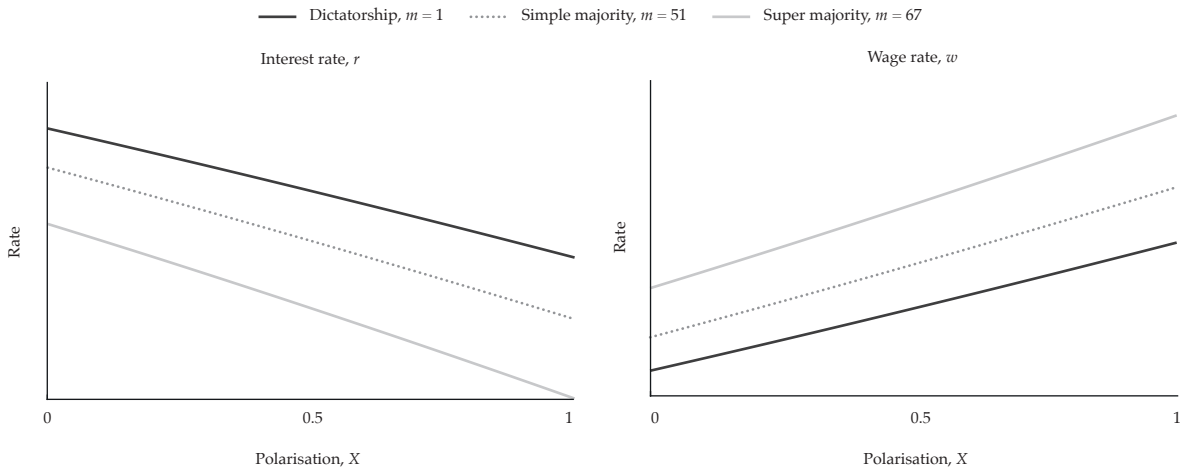
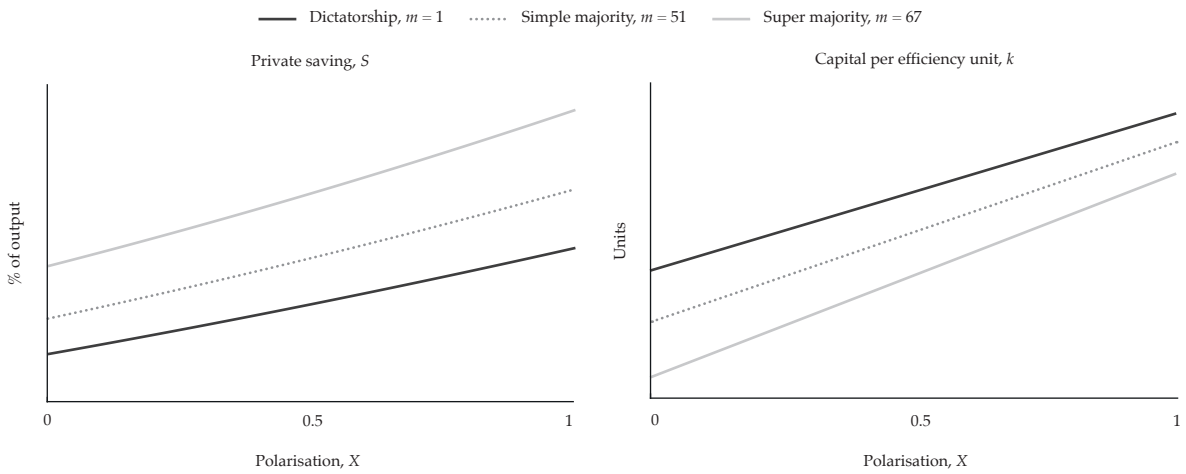
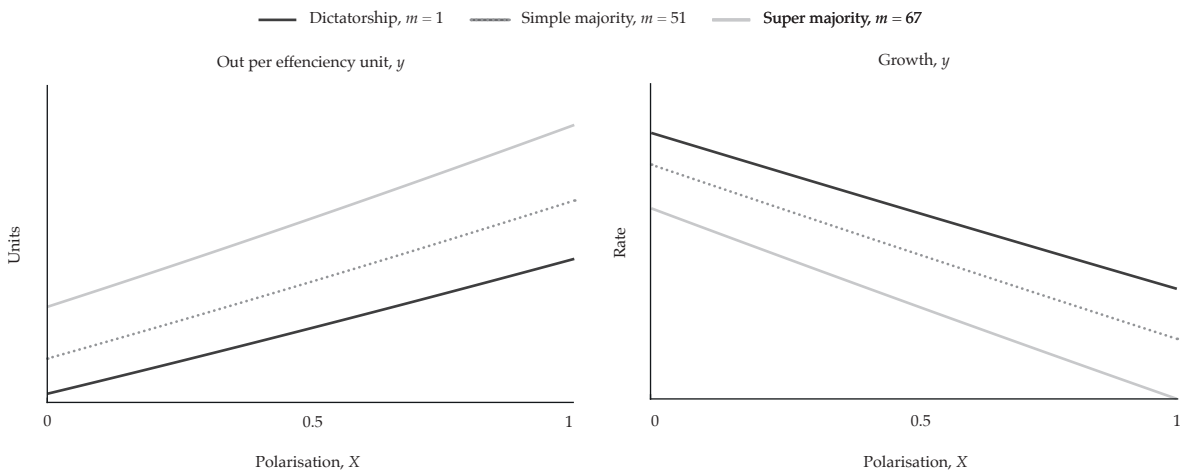


Figure 5:
Private savings S and capital per efficiency unit k



Lastly, the combined effects of lower public investment, higher private savings and a large stock of capital per efficiency unit generate two, at first contradictory, results. On one hand, the higher level of capital stock increases the balance-growth level of output per efficiency unit y, which is also higher under more democratic rules. On the other hand, long-run growth levels are decreasing in polarisation, and are lower under more democratic voting rules (see figure 6). The dynamics of growth are explained because, under a balanced-growth path, productivity growth directly depends on the stock of public investment $\gamma=(I_s\, y_s\,)^{\varphi}$ and thus it is lower under higher levels of polarisation. However, under less democratic voting rules, capital per efficiency unit is higher due to a weaker *crowding-out* effect (since public investment is lower), and thus under a steady state, more democratic economies have a higher level of output per efficiency unit but will experience slower growth.

Figure 6:
Private savings *S* and capital per efficiency unit *k*



Demand for pork barrel for the proposer's district is the highest in a dictatorship $m = 1$, and public investment is highest under democratic systems of government $m \in [1,99]$. Tax rates are also lower in a democratic government when compared to a dictatorship. These results stem from the negative effect which tax rates have on savings and thus future paths of consumption. Therefore, under democratic voting rules, tax rates are lower since legislators anticipate that higher taxes reduce savings and thus consumption. Similarly, it can be interpreted as taxes being hardest to raise when polarisation is high and agents do not internalise the need for public investment financed through higher taxes.

Public investment generates both *crowding-in* and *crowding-out* mechanisms. On one hand, both public and private investment are higher under democratic voting rules than under a dictatorship. This is accomplished by lower demand for pork barrel and lower tax rates, which push down interest rates and consumption under democracy. On the other hand, greater public investment under democracy leads to lower steady-state capital levels. This is evidence of a *crowding-out* mechanism by which greater public

investment is accompanied by lower levels of steady-state capital. Overall, at low levels of polarisation, total investment and growth are the highest.

A. Welfare analysis

Using the sequential definition of utility 5, in a balanced growth path, the productivity-adjusted variables are constant at their steady-state value, and the variables as a share of output remain constant. Hence, lifetime utility can be expressed as a function of steady-state levels of output and policy functions. In this model, under the Symmetric Markov Equilibrium, both public investment and pork barrel decisions are taken independently of the level of productivity, as shown earlier. Therefore, under balanced growth, their levels as a share of output remain constant, and the intertemporal discounting yields the lifetime utility function.

Proposition 3. *The function W expresses the lifetime utility of a representative citizen given an initial level of productivity* z_0*:*

$$\left\{\begin{array}{c} n\left(\frac{1}{1-\beta}\right)\left(\ln\left(\hat{c}_s\right)+\omega\ln(y_s)+\ln(z_0)+\varphi\left(\frac{\beta}{1-\beta}\right)\ln(I_s y_s)+\right. \\ \left.\left(\frac{1}{1-\beta}\right)\left(\omega\chi\left(\ln(G)+(m-1)\ln(G_m)+(n-m)\ln\left(\underline{G}\right)\right)+n(1-\chi)\ln\left(G^T\right)\right)\right\} \end{array} \quad (14)$$

Subsection A5. of the appendix contains the proof of the proposition 3.

Aggregate welfare is decreasing in polarisation, yet a threshold exists after which aggregate welfare under democratic voting rules falls below the aggregate welfare under a dictatorship. Figure 7 shows on the left panel the aggregate welfare of the economy under four different voting rules, and the right-hand side panel compares the level of aggregate *W* between the super and simple majority cases versus the dictatorial voting rules. There are three main results from the welfare analysis:

First, *W* is decreasing with the levels of polarisation. Both panels show that as polarisation grows, *W* falls under the four different voting rules. Because higher polarisation increases the demand for pork barrel, public investment and productivity growth falters. Lower public investment reduces growth, and the *crowding-in* effect is not sufficiently strong to counteract the fall in productivity growth. Hence, steady-state levels of consumption fall and thus utility is highest when polarisation is low because agents internalise the benefits of growth in electoral districts different to its own, as shown on the left-hand side of figure 7.

Second, aggregate welfare is the highest under unanimous voting rules. Under unanimous voting rules, the proposer needs to internalise the utility of all $m = 100$ electoral districts. Therefore, the marginal value of public investment is highest since one unit of public investment benefits all districts equally. Therefore, rather than offering pork barrel to all districts, under unanimous voting rules, the proposer chooses public investment as a means to maximise the lifetime utility of all households.

The positive externality of public investment drives the lifetime utility effect. In the long run, all districts face the same probability distribution of being part of the winning coalition, the opposition, or being the agenda setter. Lower public investment dominates the effect of lower tax rates because it is productivity growth and not capital accumulation that drives growth in this model. Even though higher levels of private capital allow greater levels of consumption, in the long run, it is through public investment that lifetime consumption can be increased the most.

However, once voting rules are not unanimous, the proposer does not need to maximise the lifetime utility of all districts but only of those needed to form

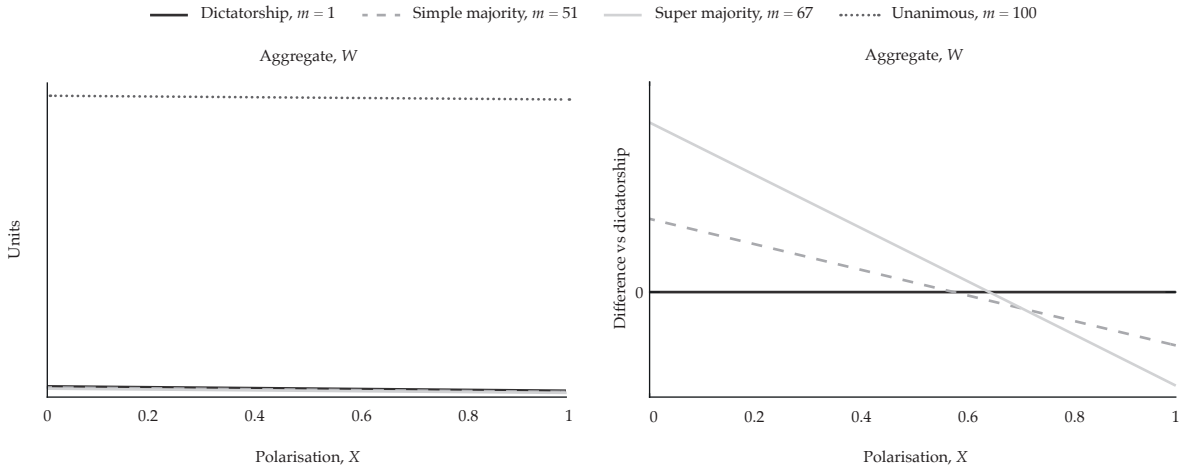
a majority. Every unit of output gained through taxes can either be used to finance public investment, which benefits all districts equally. In contrast, the same unit of tax revenues can be used to target specific public goods and marginally increase the chances of the legislature of approving the proposer's platform. Therefore, once unanimous voting rules are dropped, pork barrel gains attractiveness as a means to guarantee the support of the minimal winning coalition.

Third, a threshold exists at which higher levels of polarisation increase the attractiveness of dictatorships and lower the demand for democratic voting rules. The right-hand side panel of figure 7 compares aggregate utility levels versus the ones obtained under a democracy (in essence, a transformation of the left-hand side). Below the threshold, the aggregate utility is highest under a super majority. However, as polarisation increases, so does the demand for pork barrel.

The more stringent the majority requirements to vote in favour of a policy platform (higher $m < 100$), the higher the total demand for pork barrel and thus the lower the public investment will be. Since $m > 1$ districts are needed to approve the policy platform, total spending on public goods increases. In contrast, under a dictatorship, even though the dictator's electoral district will enjoy a much larger value of public goods, total spending will tend to fall. Therefore, the *crowding-in* effects of public spending are weaker and aggregate utility falls sharply under a democracy when polarisation increases.

Although households and legislators in this model are forward-looking, polarisation alters the extent to which future investment benefits are internalized in the political bargaining process. Higher polarisation weakens legislators' concern for aggregate welfare, leading them to place relatively greater weight on current, district-specific transfers and less on future returns from public investment that accrue broadly across districts. This bias does not stem from myopic preferences but from the equilibrium outcome of a political process in which each legislator internalizes only a fraction of the common gains from investment. In a limiting case of low polarisation, where legislators' preferences are fully aligned, agents would internalise the future expansion of the tax base, and the bias toward pork-barrel expenditure would vanish.

Figure 7:
Aggregate welfare under different voting rules



An additional result related to the way the political process works stems from the range in which regime switching can occur. The right-hand panel of figure 7 shows a range of χ in which, under a dictatorship, a super majority is preferable but not a simple majority. This intermediate level of polarisation implies that under a simple majority, slightly less than half of the legislature are not members of the winning coalition and hence do not enjoy the benefits of pork barrel. In contrast, under a super majority, the fraction of electoral districts left out of the distribution of pork barrel is smaller.

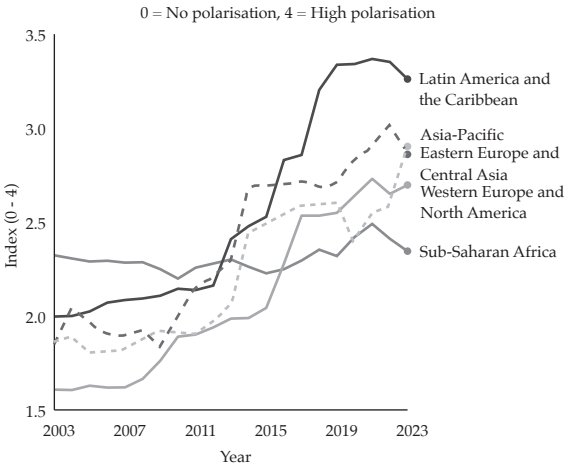
Overall, the results from this model suggest that greater polarisation lowers growth through a higher demand for pork barrel and lower public investment. Moreover, the rules of the voting system affect the results. Under democratic voting systems, tax rates, growth, and consumption are lower, yet wages and private savings are higher. Lastly, under lower levels of polarisation, aggregate welfare is greater under democracy than dictatorship, yet it is always decreasing as polarisation rises.

V. Discussion

A. Polarisation and growth trends in the real world

Stylised facts from across different world regions suggest a link between political polarisation and economic growth. Using data from the IMF's World Economic Outlook for GDP growth (*Steady but Slow: Resilience amid Divergence* 2024) and from the Varieties of Democracy Institute (Coppedge et al. 2024), Figures 8 and 9 illustrate two key patterns: political polarisation has risen steadily across world regions, while average economic growth has declined since 2013. These trends motivate the theoretical model developed in this paper. Within the model, polarisation operates through the spillover parameter χ, but its implications extend beyond a mechanical reduction in the value of public investment. A lower χ weakens legislators' incentives to internalise the collective returns from taxation and investment, which can reduce equilibrium tax revenues and total fiscal expenditure. The primary effect, however, is compositional: as legislators become more locally oriented, fiscal resources shift away from public investment—whose benefits are broadly shared—toward district-targeted spending that yields immediate, localised gains.

Figure 8:
Political polarisation is on the rise across different regions



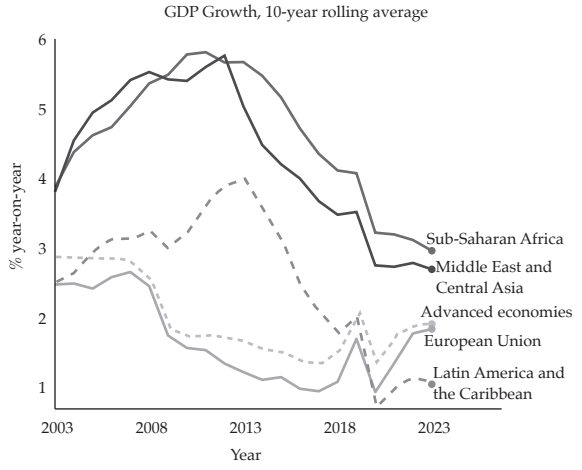
First, polarisation is on the rise across different economic regions. Second, growth rates have continuously slowed down since the 2000s. In the case of individual regions, the cases of Sub-Saharan Africa and Latin America and the Caribbean stand out as illustrative examples[3]. On one hand, as shown in figure 8, polarisation is the highest in Latin America, just as it has increased the most during the last twenty years. During the same time frame, growth initially increased during the 2000s, yet by 2023, average growth is the lowest across the studied regions. On the other hand, polarisation was initially the highest in Sub-Sa-

3 The name of the regions varies across both figures due to different naming conventions used by both sources.

haran Africa, yet compared with the other world regions, it remained relatively stable. Twenty years later, polarisation in Sub-Saharan Africa remained relatively stable and below the levels seen by different country groups. GDP growth for Sub-Saharan Africa began sliding down after 2013, yet by 2023 it remained above the levels of the rest of the country groups.

Both results illustrate the use of the theoretical model developed here as a tool to link polarisation and growth through the fiscal policy channel. The stylised facts presented here are not indicative of complete and direct causality between both variables, yet the model presented here allows for interpreting lower growth through the lens of political polarisation. This study contributes to existing literature strands that connect growth with political economy through the institutional channel, in this case, through the formation of fiscal policy. These stylised facts motivate the theoretical framework I create in this paper that shows one possible mechanism through which higher levels of polarisation drive lower growth rates. Therefore, the natural extension to this paper in future research would be the empirical verification of the key results derived from my model.

Figure 9:
Average GDP growth is slowing across different regions



B. Discussion of modelling choices

This model contributes to the new political economy literature in three main dimensions. First, it extends the *bargaining in a legislature* framework of Baron and Ferejohn (1989), Battaglini and Coate (2007) and Battaglini and Coate (2008) by introducing physical capital accumulation and productivity growth into what were originally labour-only economies. This addition generates a second externality: the positive effect of public investment on productivity growth. Second, the model simplifies the growth process relative to Barseghyan and Battaglini (2016), by assuming a single source of productivity growth and excluding debt financing. This simplification allows me to isolate the key trade-off between the distributive spillovers of pork barrel and the *crowding-in* externality of public investment. Third, it embeds the legislative bargaining process into a time-inconsistent fiscal policy environment à la Klein et al. (2008), where policymakers cannot commit to future policies and face periodic replacement.

The theoretical framework used in this model relies on four simplifying assumptions that clarify the mechanism through which polarisation affects fiscal outcomes. First, public investment is treated as a pure public good benefiting all districts equally, while district-specific goods do not contribute to productivity growth[4]. These assumptions position the model as an extreme case that captures the core tension between the growth-enhancing role of capital expenditure and the distributive incentives of current expenditure. Extending the model to allow district-specific productive goods would make it possible to analyse economic inequality between districts as an additional source of polarisation.

While the model defines pork-barrel expenditure as district-specific public goods, this representation

4 In reality, local public goods such as schools or infrastructure may enhance productivity. This model abstracts from such effects to isolate the fiscal trade-off between productive and unproductive expenditure.

should be interpreted more broadly. The distinction across "districts" is a useful way to model politically targeted spending, encompassing not only geographically bounded allocations but also sectoral subsidies, partisan transfers, and networks of clientelism. The essential feature remains in which fiscal resources are fragmented across identifiable constituencies, reflecting political incentives to direct spending toward particular groups, and thus districts need not to be identified as geographically bounded electoral districts.

Second, agents are homogeneous and share complete information. Following the new political economy tradition of Alesina and Tabellini (1990) and Tabellini and Alesina (1990), these assumptions imply that conflicts of interest, rather than informational asymmetries, drive fiscal outcomes. They place the model within the rational-expectations family, contrasting with the political business cycle models of Nordhaus (1975) and Fergusson and Suárez (2016). As Snowdon, vane, and Alesina (1999) observes, in this tradition "conflict of interest is much more important than asymmetric information". Homogeneous districts ensure that all agents are ex-ante identical, which permits a single value function to summarise the entire equilibrium. Complete information implies that sub-optimal outcomes emerge from strategic interaction among rational actors rather than uncertainty or misperception.

Third, the model embeds several externalities that interact with these assumptions. The first, central to the political conflict, is the positive spillover of pork barrel. Increases in $\mathcal{G}_i$ raise the contemporaneous utility of households across districts, even when spending is targeted at the minimum winning coalition. When polarisation is low, these spillovers are stronger, generating incentives for the proposer to expand pork barrel. The second externality is the growth-enhancing effect of public investment $\mathcal{I}$, which raises productivity and future welfare, thereby amplifying the long-term benefits of cooperative fiscal behaviour. Finally, the tax rate τ creates a negative externality by reducing private savings and, consequently, future fiscal capacity.

Lastly, the model is solved as a simultaneous game between market participants and the legislature. Households optimise consumption and savings given fiscal policy, while legislators determine fiscal instruments without internalising the households' reaction functions. This approach preserves analytical tractability and captures the spirit of the *public choice* tradition, where policymakers are themselves voters and consumers but collective decisions arise from strategic rather than coordinated behaviour.

Taken together, these modelling choices serve two purposes. They sharpen the focus on how polarisation and legislative bargaining distort the allocation of public expenditure, and they anchor the model within the comparative political economy literature that emphasises institutional incentives over informational frictions.

VI. Conclusion

The effects of polarisation on investment and growth are negative and non-linear, with higher polarisation being associated with lower growth. The model here developed introduced private capital into the labour-only legislative bargain economy of Battaglini and Coate (2008). This modification allows for the introduction of both *crowding-in* and *crowding-out*, which both amplify or mitigate the effects of polarisation. The main results of this paper show how under higher levels of political polarisation, the provision of public investment falls and the provision of current government expenditure increases as a share of the budget.

The model shows how increasing levels of polarisation lead agents to internalise less the utility derived from public spending by other agents. Higher polarisation thus increases demand for targeted public goods for each electoral district rather than public investment, which improves the outcomes for all districts in equal fashion. It is due to political conflict that the demand for pork barrel increases as demand

for public investment falls. Since pork barrel is only offered to the winning coalition members, the total financing needs fall, and thus, with higher polarisation, tax rates tend downwards.

This shift in the composition of fiscal policy, as polarisation increases (higher demand for unproductive public spending, lower public investment, and lower tax rates), generates two seemingly contradictory effects. First, lower taxes encourage greater private savings, which increases capital levels and thus output. Second, lower public investment reduces growth rates and thus both *crowding-in* and *crowding-out* mechanisms balance each other out.

This model further allows the analysis of welfare and market dynamics under different voting rules. When voting rules are democratic and a majority of votes is needed to approve a fiscal policy platform, wage rates and output tend to be higher, yet growth and the demand for pork barrel for the proposer's district are lower. This is balanced by lower tax rates under democratic voting rules, which induce higher aggregate investment (both public and private). Lastly, the model suggests that a very high level of polarisation agents perceive a high lifetimeime utility under autocratic voting rules.

Empirical validation of the different mechanisms and results of this model opens up a rich research agenda. For instance, comparing periods of high and low political polarisation within countries would be an interesting area of research. The stylised facts highlighted in the introduction yield some motivation to study the co-movements of political polarisation and growth. For example, in my model, higher wages and capital stock are associated with higher levels of polarisation. One interesting avenue would be to study the prevalence of semantically-linked mentions of public investment vs current expenditure in the political manifestos of parties engaged in close-call elections. Close elections serve as a proxy of a natural experiment, and the use of text mining would allow to reveal a latent variable found in the political manifestos, or even candidate speeches.

A second avenue of research would be to further study the inverse causality, which this paper does not discuss. It is also possible that low growth or other economic dynamics increase political polarisation. Gabriel, Klein, and Pessoa (2023) present evidence that austerity measures in the European Union during the early 2010s were associated with increases in polarisation and extreme parties' voter share. Further exploring the links between low growth and political polarisation is also relevant o to untangle the political economy of growth.

In terms of normative implications, this model highlights the importance that achieving low levels of polarisation has on improving growth. In the real world, governments interested in promoting public investment would find it easier to see their budgets approved when polarisation was lower. Promoting social exchanges between members of different political camps would help to reduce the separating effect of polarisation. Open-access public buildings that offer services to a wide targets and not just tailored to specific interest groups would introduce spaces where individuals interact with each other, essentially lowering the parameter χ of my model.

Overall, lower levels of polarisation improve aggregate welfare, growth, and the different metrics studied here. Therefore, the main normative implication is a call for civil actors to reduce polarisation levels because lower levels of it are associated with greater levels of welfare. In this model, doing so is not in the interest of policymakers since higher polarisation allows them to increase their share of pork barrel. This model does not study the social origins of polarisation, and thus an interesting scope for study goes the other way: how does fiscal policy affect polarisation levels. This model allows the study of how polarisation affects economic and welfare variables through the fiscal policy channel.

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VIII. Appendix

Derivations and proofs

A1. Market equilibrium

A1.a. Solution to the household's problem

Consider the Lagrangian $\mathcal{L}$:

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t \left(\ln(c_t)\right) + \lambda_t(w_t l_t + (1 + (1 - \tau_t)r_t)a_t - c_t - a_{t+1})$$

Let the first order conditions be $\frac{\partial \mathcal{L}}{\partial x_t} = 0$ for $x_t \in \{c_t, l_t, a_{t+1}, \lambda_t\}$ for :

Consumption C_t: $\beta^t = \lambda_t c_t$

Private savings a_{t+1}: $\lambda_t = \lambda_{t+1}(1 + (1 - \tau_{t+1})r_{t+1})$

Lagrange multiplier λ_t: $c_t + a_{t+1} = w_t l_t + (1 + (1 - \tau_{t+1})r_{t+1})a_t$

Iterating the consumption FOC one step ahead and combining it with the private savings FOC yields the euler equation shown in 6. The FOC for the lagrange multiplier represents the budget condition in 7.

A1.b. Equations that determine the market equilibrium given a fixed fiscal policy

Combining the different conditions with the set of endogenous variables are: t $\{c_t, l_t, a_{t+1}, K_t, L_t, w_t, r_t, z_t, I_t, B_{t+1}\}$ he market equilibrium can be summarised by the following equations:

Government budget restriction: $I_t + \sum_{i=1}^{n} \mathcal{G}_{i,t} = \tau_t r_t \sum_{i=1}^{n} a_t$

Productivity growth: $\frac{z_{t+1}}{z_t} = \left(\frac{\mathcal{I}_t}{z_t}\right)^{\varphi}$

Euler' s equation: $\frac{c_{t+1}}{c_t} = \beta(1 + (1 - \tau_{t+1})r_{t+1})$

Interest rate: $r_t = \alpha \frac{Y_t}{K_t} - \delta$

Wage rate: $w_t = (1 - \alpha)\frac{Y_t}{L_t}$

Labour market: $N = L_t$

Capital market: $na_{t+1} = K_{t+1}$

Private investment: $S_{t+1} = K_{t+1} - (1-\delta)K_t$

Final good: $n\, c_t + K_{t+1} - (1-\delta)K_t + \mathcal{I}_t + \sum_{i=1}^{n} \mathcal{G}_{i,t} = Y_t$

Capital dynamics: $\frac{K_{t+1}}{K_t} = \frac{Y_t}{K_t} + (1-\delta) - \frac{1}{K_t}\left(\mathcal{I}_t + \sum_{i=1}^{n} \mathcal{G}_{i,t}\right) - \frac{n\, c_t}{K_t}$

A2. Indirect Utility functions

Proof to proposition 1

Proof. The household's utility function is:

$$\mathcal{U}_i(c_t, l_t) = \ln(c_t) + \omega \ln\left(\mathcal{G}_{i,t}^{\chi}\left(\sum_{j=1}^{n} \mathcal{G}_{j,t}\right)^{1-\chi}\right)$$

With the final good market clearing condition 10, in equilibrium C_t can be expressed as:

$$n\, c_t + K_{t+1} - (1-\delta)K_t + \mathcal{I}_t + \sum_{i=1}^{n} \mathcal{G}_{i,t} = Y_t$$

$$n\, c_t = Y_t - K_{t+1} + (1-\delta)K_t - \mathcal{I}_t - \sum_{i=1}^{n} \mathcal{G}_{i,t}$$

$$n\, c_t = Y_t - \mathcal{S}_{t+1} - \mathcal{I}_t - \sum_{i=1}^{n} \mathcal{G}_{i,t}$$

$$\frac{n\, c_t}{Y_t} = 1 - S_{t+1} - I_t \sum_{i=1}^{n} G_{i,t}$$

Now, C_t can be further simplified by:

$$C_t(S_{t+1}, I_t, G_t) = \frac{n\, c_t}{Y_t} \Longrightarrow c_t = \frac{C_t(S_{t+1}, I_t, G_t)Y_t}{n}$$

Therefore, the household's utility function can be rearranged as:

$$\mathcal{U}_i(c_t, l_t) = \ln\left(Y_t - \mathcal{S}_{t+1} - \mathcal{I}_t - \sum_{i=1}^{n} \mathcal{G}_{i,t}\right) - \ln(n) + \omega \ln\left(\mathcal{G}_{i,t}^{\chi}\left(\sum_{j=1}^{n} \mathcal{G}_{j,t}\right)^{1-\chi}\right)$$

$$= \ln(Y_t) + \ln\left(1 - S_{t+1} - I_t - \sum_{i=1}^{n} G_{i,t}\right) - \ln(n) + \omega \ln\left((Y_t G_{i,t})^{\chi}\left(Y_t \sum_{j=1}^{n} G_{j,t}\right)^{1-\chi}\right)$$

$$= (1+\omega)\ln(Y_t) + \ln\left(C_t(\mathcal{S}_{t+1}, \mathcal{I}_t, \mathcal{G}_t)\right) - \ln(n) + \omega \ln\left((G_{i,t})^{\chi}\left(\sum_{j=1}^{n} G_{j,t}\right)^{1-\chi}\right) \text{ (Factor } Y_t)$$

$$= (1+\omega)\ln(Y_t) + \ln\left(\frac{C_t(\mathcal{S}_{t+1}, \mathcal{I}_t, \mathcal{G}_t)}{n}\right) + \omega \ln\left((G_{i,t})^{\chi}\left(\sum_{j=1}^{n} G_{j,t}\right)^{1-\chi}\right)$$

Since $\mathcal{S}_{t+1} = \mathcal{K}_{t+1} - (1-\delta)\mathcal{K}_t$, then:

$$S_{t+1} = \frac{\mathcal{K}_{t+1}}{Y_t} - (1-\delta)\frac{\mathcal{K}_t}{Y_t}$$

$$= \frac{z_{t+1}k_{t+1}}{z_t y_t} - (1-\delta)\frac{z_t k_t}{z_t y_t}$$

$$= \frac{z_{t+1}}{z_t}\left(\frac{\mathcal{H}(k_t, G_t, I_t)}{y_t}\right) - (1-\delta)\frac{k_t}{y_t}$$

$$= \left(\frac{\mathcal{I}_t}{z_t}\right)^{\varphi}\left(\frac{\mathcal{H}(k_t, G_t, I_t)}{y_t}\right) - (1-\delta)\frac{k_t}{y_t}$$

$$= \left(\frac{Y_t I_t}{z_t}\right)^{\varphi}\left(\frac{\mathcal{H}(k_t, G_t, I_t)}{y_t}\right) - (1-\delta)\frac{k_t}{y_t}$$

$$= (y_t I_t)^{\varphi}\left(\frac{\mathcal{H}(k_t, G_t, I_t)}{y_t}\right) - (1-\delta)\frac{k_t}{y_t}$$

Therefore, $S_{t+1} \equiv S_{t+1}(k_t, y_t(k_t), \mathcal{H}(k_t, G_t, I_t), I_t, G_t)$. However $y_t = k_t^{\alpha} L^{1-\alpha}$, and $z_{t+1} = (I_t y_t)^{\varphi} z_t$. Therefore, $S_{t+1} \equiv S_{t+1}(k_t, I_t, G_t)$. Hence, $C_t \equiv C_t(k_t, I_t. G_t)$ and consumption as a share of total output depends only on the state variable k_t (productivity adjusted capital) and the shares of output devoted to public goods and public investment . Therefore, the indirect utility function can be rearranged as:

$$\mathcal{U}_i(c_t, l_t) = u(k_t, G_t, I_t) + (1+\omega)\ln(Y_t) + \omega\ln\left(G_{i,t}^{\chi}\left(\sum_{j=1}^{n} G_{j,t}\right)^{1-\chi}\right)$$

A3. Balanced growth path given a fixed fiscal policy

Suppose that a balanced growth path (BGP) exists and that a stable fiscal policy platform $\mathcal{P}_s \equiv \{\tau_s, I_s, G_{1,s}, \dots, G_{n,s}\}$ also exists such that the economy remains stable growing at a constant rate. Define $\widehat{c_t} = \frac{c_t}{z_t}$ as productivity-adjusted consumption. This is an even more difficult variable to interpret yet it will allow to solve for the BGP. The definition of a BGP implies that: $\frac{k_{t+1}}{k_t} = 1, \frac{y_{t+1}}{y_t} = 1,$ and $\frac{\hat{c}_{t+1}}{\hat{c}_t} = 1.$ and

Using these results, the **households euler equation** in a balanced growth depends on the level of productivity and public investment. Hence,

$$\frac{c_{t+1}}{c_t} = \beta(1 + (1-\tau_s)r_s)$$

$$\frac{z_{t+1}}{z_t}\frac{\hat{c}_{t+1}}{\hat{c}_t} = \beta\left(1 + (1-\tau_s)\left(\alpha\frac{y_s}{k_s} - \delta\right)\right)$$

$$(I_s y_s)^{\varphi} = \beta\left(1 + (1-\tau_s)\left(\alpha\frac{y_s}{k_s} - \delta\right)\right)$$

$$\frac{(I_s y_s)^{\varphi}}{\beta} - 1 = (1-\tau_s)\left(\alpha\frac{y_s}{k_s} - \delta\right)$$

$$\frac{\frac{(I_s y_s)^{\varphi}}{\beta} - 1}{\alpha(1-\tau_s)} = k^{\alpha-1}L^{1-\alpha} - \delta$$

$$\left[\frac{\frac{(I_s y_s)^{\varphi}}{\beta} - 1}{\alpha(1-\tau_s)} + \delta\right] L^{1-\alpha} = k^{\alpha-1}$$

$$\left[\frac{\frac{(I_s y_s)^{\varphi}}{\beta} - 1 + \delta\alpha(1-\tau_s)}{\alpha(1-\tau_s)}\right] L^{1-\alpha} = k^{\alpha-1}$$

Therefore, in a balanced growth path, the stationary level of productivity-adjusted capital is defined implicitly by the following function:

$$k_s = \left(\frac{\alpha\beta(1-\tau_s)}{(I_s y_s)^{\varphi} - \beta\left(1 - \delta\alpha(1-\tau_s)\right)}\right)^{\frac{1}{1-\alpha}} L$$

where productivity-adjusted production is

$$y_s = k_s^{\alpha} L^{1-\alpha}.$$

Using these results and the dynamic of capital accumulation:

$$\frac{z_{t+1}}{z_t}\frac{k_{t+1}}{k_t} = \frac{y_t}{k_t} + (1-\delta) - \frac{y_t}{k_t}\left(I_t + \sum_{i=1}^{n} G_{i,t}\right) - \frac{n\,\hat{c}_t}{k_t}$$

$$(I_s y_s)^{\varphi} = \frac{y_s}{k_s} + (1-\delta) - \frac{y_s}{k_s}\left(I_s + \sum_{i=1}^{n} G_{i,s}\right) - \frac{n\,\hat{c}_s}{k_s}$$

$$n\,\hat{c}_s = y_s + (1-\delta)k_s - y_s\left(I_s + \sum_{i=1}^{n} G_{i,s}\right) - (I_s y_s)^{\varphi} k_s$$

Therefore, in a balanced growth path, the stationary level of the balanced budget becomes:

$$y_s\left(I_s + \sum_{i=1}^{n} G_{i,s}\right) = \tau_s(\alpha y_s - \delta k_s)$$

Therefore, since for a BGP to exist the relation $I_s\, y_s$ needs to be a constant, then I_s also needs to be stable in a BGP. The same applies for τ_s. Lastly, the distribution of public goods does not impact the steady-state levels as long ad the total amount of public goods remain the same.

Definition 1 (Balanced growth path). *A balanced growth path in this economy occurs when all endogenous variables grow at the same rate as productivity grows, and this occurs at at constant rate. Let* $\gamma = \frac{z_{t+1}}{z_t}$ *be a constant. Then, the economy is at a balanced growth path when* $\forall x \in \{c, \mathcal{K}, Y\}$ *it occurs that* $\frac{x_{t+1}}{x_t} = \gamma$.

I need to prove that in a BGP, the ratios $\frac{c}{Y}, \frac{\mathcal{K}}{Y}, \frac{\mathcal{I}}{Y}$ are all constant.

Assume that productivity growth is a constant $\gamma = \frac{z_{t+1}}{z_t}$:

A3.a. Capital

In a BGP, capital growth occurs and is such that $\frac{\mathcal{K}_{t+1}}{\mathcal{K}_t} = \frac{z_{t+1}}{z_t}$.

Therefore:

$$Y_t = K_t^{\alpha}(z_t L_t)^{1-\alpha}$$

$$= z_t^{1-\alpha} K_t^{\alpha} L_t^{1-\alpha}$$

$$\frac{Y_{t+1}}{Y_t} = \left(\frac{z_{t+1}}{z_t}\right)^{1-\alpha}\left(\frac{K_{t+1}}{K_t}\right)^{\alpha}\left(\frac{L_{t+1}}{L_t}\right)^{1-\alpha}$$

$$\frac{Y_{t+1}}{Y_t} = \left(\frac{z_{t+1}}{z_t}\right)^{1-\alpha}\left(\frac{z_{t+1}}{z_t}\right)^{\alpha}$$

$$\frac{Y_{t+1}}{Y_t} = \left(\frac{z_{t+1}}{z_t}\right)$$

Therefore, under a BGP. $\frac{\mathcal{K}_{t+1}}{Y_{t+1}} = \frac{\mathcal{K}_t}{Y_t}$.

A3.b. Euler's

$$\frac{c_{t+1}}{c_t} = \beta(1 + (1-\tau_s)r_s)$$

$$\frac{z_{t+1}}{z_t}\frac{\hat{c}_{t+1}}{\hat{c}_t} = \beta\left(1 + (1-\tau_s)\left(\alpha\frac{y_s}{k_s} - \delta\right)\right)$$

$$(I_s y_s)^{\varphi} = \beta\left(1 + (1-\tau_s)\left(\alpha\frac{y_s}{k_s} - \delta\right)\right)$$

If productivity growth is constant, then $\gamma = \left(\frac{\mathcal{I}_t}{z_t}\right)^{\varphi}$.

$$\gamma = \left(\frac{\mathcal{I}_t}{z_t}\right)^{\varphi}$$

$$\gamma = \left(\frac{I_t Y_t}{z_t}\right)^{\varphi}$$

$$\gamma = (I_t y_t)^{\varphi}$$

$$I_t = \frac{\gamma^{\frac{1}{\varphi}}}{y_t}$$

A3.c. Capital

In a BGP, capital growth occurs and is such that $\frac{\mathcal{K}_{t+1}}{\mathcal{K}_t} = z$.

A4. Political equilibrium

Proof of proposition 1.

Proof. The proof involves applying the expected value to the term which contains the utility of public goods, an subtracting terms from both sides of the equation.

- Suppose that the incentive compatibility constraint is satisfied with equality. Then, equal terms are subtracted from both sides using the definition of $V_{\mathcal{C}}(b,k,z)$

$$u(k)+(1+\omega)\ln(zy)+\omega\ln\left(G_m^{\chi}\left(\sum_{j=1}^{n}G_j\right)^{1-\chi}\right)+\beta\mathbb{E}[V(k',z')]=V_{\mathcal{C}}(k,z)$$

$$\omega\ln\left(G_m^{\chi}\left(\sum_{j=1}^{n}G_j\right)^{1-\chi}\right)=\omega\mathbb{E}\left[\ln\left(G_i^{\chi}\left(\sum_{j=1}^{n}G_j\right)^{1-\chi}\right)\right]$$

Then, the expected value is expanded and the externality of total public goods provision is cancelled out.

$$\chi\omega\ln(G_m)+(1-\chi)\omega\ln\left(\sum_{j=1}^{n}G_j\right)\&=\chi\omega\mathbb{E}[\ln(G_i)]+(1-\chi)\omega\mathbb{E}\left[\ln\left(\sum_{j=1}^{n}G_j\right)\right]$$

Since the game is symmetric, the expected value of the sum of district-specific public goods is the same. Then, the expected value of the public good which legislative member *i* is applied

$$\begin{aligned}
\ln(G_m)&=\mathbb{E}[\ln(G_i)]\\
\ln(G_m)&=\frac{1}{n}\big(\ln(G)+(m-1)\ln(G_m)+(n-m)\ln(\underline{G})\big)\\
n\ln(G_m)&=\ln(G)+(m-1)\ln(G_m)+(n-m)\ln(\underline{G})\\
(n-m+1)\ln(G_m)&=\ln(G)+(n-m)\ln(\underline{G})\\
\ln(G_m)&=\frac{\ln(G)}{(n-m+1)}+\frac{n-m}{n-m+1}\ln(\underline{G})\\
\ln(G_m)&=\ln\left(G^{\frac{1}{n-m+1}}\underline{G}^{\frac{n-m}{n-m+1}}\right)\\
G_m&=G^{\frac{1}{n-m+1}}\underline{G}^{1-\frac{1}{n-m+1}}
\end{aligned}$$

- The proof in the opposite direction is analogous.

The expected value of receiving public goods at the start of each round becomes:

$$\mathbb{E}[\ln(G_i)]|_{i\in\{1,\dots,n\}} = \frac{1}{n}\left(\ln(G) + (m-1)\ln(G_m) + (n-m)\ln\left(\underline{G}\right)\right)$$

$$\mathbb{E}[\ln(G_i)]|_{i\in\{1,\dots,n\}} = \frac{1}{n}\left(\ln(G) + (m-1)\ln\left(G^{\frac{1}{n-m+1}}\underline{G}^{1-\frac{1}{n-m+1}}\right) + (n-m)\ln\left(\underline{G}\right)\right)$$

$$\mathbb{E}[\ln(G_i)]|_{i\in\{1,\dots,n\}} = \frac{1}{n}\left(\ln(G) + \frac{m-1}{n-m+1}\ln(G) + (m-1)\ln\left(\underline{G}^{1-\frac{1}{n-m+1}}\right) + (n-m)\ln\left(\underline{G}\right)\right)$$

$$\mathbb{E}[\ln(G_i)]|_{i\in\{1,\dots,n\}} = \frac{1}{n}\left(\frac{n}{n-m+1}\ln(G) + (m-1)\ln\left(\underline{G}^{1-\frac{1}{n-m+1}}\right) + (n-m)\ln\left(\underline{G}\right)\right)$$

$$\mathbb{E}[\ln(G_i)]|_{i\in\{1,\dots,n\}} = \frac{1}{n}\left(\frac{n}{n-m+1}\ln(G) + \left((m-1)\left(1-\frac{1}{n-m+1}\right) + (n-m)\right)\ln\left(\underline{G}\right)\right)$$

$$\mathbb{E}[\ln(G_i)]|_{i\in\{1,\dots,n\}} = \frac{1}{n}\left(\frac{n}{n-m+1}\ln(G) + \left(\frac{(n-m)(m-1)+(n-m)(n-m+1)}{n-m+1}\right)\ln\left(\underline{G}\right)\right)$$

$$\mathbb{E}[\ln(G_i)]|_{i\in\{1,\dots,n\}} = \frac{1}{n}\left(\frac{n}{n-m+1}\ln(G) + \left(\frac{(n-m)(m-1)+(n-m)(m-1-n)}{n-m+1}\right)\ln\left(\underline{G}\right)\right)$$

$$\mathbb{E}[\ln(G_i)]|_{i\in\{1,\dots,n\}} = \frac{1}{n}\left(\frac{n}{n-m+1}\ln(G) + \left(\frac{n(n-m)}{n-m+1}\right)\ln\left(\underline{G}\right)\right)$$

$$\mathbb{E}[\ln(G_i)]|_{i\in\{1,\dots,n\}} = \frac{1}{n-m+1}\ln(G) + \left(\frac{n-m}{n-m+1}\right)\ln\left(\underline{G}\right)$$

A4.a. Proof of proposition 2

Proof. Let with $\sum_{s=t}^{\infty}\beta^{s-t}\,\mathcal{U}_i(k_s,z_s)$ with $\mathcal{U}_i(k_s,z_s) = u(k_s) + (1+\omega)\ln(Y_s) + \omega\mathbb{E}\left[\ln\left(G_{i,s}^{\chi}\left(\sum_{j=1}^{n}G_{j,s}\right)^{1-\chi}\right)\right]$.

Remember that $Y_t = z_t\, y_t$. Then can be rearranged as:

$$v_t = \sum_{s=t}^{\infty}\beta^{s-t}\left\{(1+\omega)\ln(z_s) + u(k_s) + (1+\omega)\ln(y_s) + \omega\mathbb{E}\left[\ln\left(G_{i,s}^{\chi}\left(\sum_{j=1}^{n}G_{j,s}\right)^{1-\chi}\right)\right]\right\}$$

Recursively, the general representation can be found for $\sum_{s=t}^{\infty}\beta^{s-t}\,(1+\omega)\ln(z_s)$ by iterating forwards the autoregressive term in order to express the function simply in terms of present productivity and the future path of public policies.

$$\begin{aligned}\ln(z_{t+1}) &= ln\left(\mathcal{I}_t^{\varphi} z_t^{1-\varphi}\right)\\ &= \varphi\ln\left(\frac{I_t Y_t}{z_t}\right) + \ln(z_t)\\ &= \varphi\ln\left(\frac{I_t z_t y_t}{z_t}\right) + \ln(z_t)\\ &= \varphi\ln(I_t y_t) + \ln(z_t)\end{aligned}$$

Iterating a second step ahead:

$$\begin{aligned}\ln(z_{t+2}) &= \varphi\ln(I_{t+1}y_{t+1}) + \ln(z_{t+1}) \\ &= \varphi\ln(I_{t+1}y_{t+1}) + (\varphi\ln(I_t y_t) + \ln(z_t)) \\ &= \varphi\ln(I_{t+1}y_{t+1}) + \varphi\ln(I_t y_t) + \ln(z_t)\end{aligned}$$

A pattern can be conjectured in which:

$$\ln(z_{t+h}) = \ln(z_t) + \varphi \sum_{j=0}^{h-1} \ln\left(I_{t+j}y_{t+j}\right)$$

$$\ln(z_{t+h}) = \ln(z_t) + \varphi \sum_{j=t}^{t+h-1} \ln\left(I_j y_j\right)$$

$$\ln(z_{t+s}) = \ln(z_t) + \varphi \sum_{j=t}^{t+s-1} \ln\left(I_j y_j\right)$$

Therefore, the first terms of can be rearranged:

$$\begin{aligned}\sum_{s=t}^{\infty} \beta^{s-t}(1+\omega)\ln(z_s) &= (1+\omega)\sum_{s=0}^{\infty}\beta^s \ln\left(z_{t+s}\right) \\ &= (1+\omega)\sum_{s=0}^{\infty}\beta^s\left(\ln(z_t) + \varphi\sum_{j=t}^{t+s-1}\ln\left(I_j y_j\right)\right) \\ &= \frac{1+\omega}{1-\beta}\ln(z_t) + (1+\omega)\sum_{s=0}^{\infty}\beta^s\left(\varphi\sum_{j=t}^{t+s-1}\ln\left(I_j y_j\right)\right) \\ &= \frac{1+\omega}{1-\beta}\ln(z_t) + (1+\omega)\sum_{s=0}^{\infty}\beta^{s-t}\left(\varphi\sum_{j=t}^{s-1}\ln\left(I_j y_j\right)\right)\end{aligned}$$

Now, the last term is rearranged in order to obtain a simpler form which can then be used to express v_t for the proof.

$$\sum_{s=t}^{\infty}\beta^{s-t}\left(\varphi\sum_{j=t}^{s-1}\ln\left(I_j y_j\right)\right)$$

Let $\Delta_0 = \beta^0\varphi\ln(I_{t-1}y_{t-1})$. First iteration with $s = t + 1$:

$$\Delta_1 = \beta^{t+1-t}\left(\varphi\sum_{j=t}^{t+1-1}\ln\left(I_j\, y_j\right)\right)\& = \beta\varphi(\ln(I_t\, y_t))$$

Second iteration with $s = t + 2$:

$$\Delta_2 = \beta^2\left(\varphi\sum_{j=t}^{t+1}\ln\left(I_j\,y_j\right)\right)\& = \beta^2\varphi(\ln(I_t\,y_t) + \ln(I_{t+1}\,y_{t+1}))$$

Third iteration with $s = t + 3$:

$$\Delta_3 = \beta^3\left(\varphi\sum_{j=t}^{t+2}\ln\left(I_j\,y_j\right)\right)\& = \beta^3\varphi(\ln(I_t\,y_t) + \ln(I_{t+1}\,y_{t+1}) + \ln(I_{t+1}\,y_{t+1}))$$

Therefore:

$$\begin{aligned}\sum_{s=t}^{3}\Delta_{s+1} &= \varphi\big((\beta+\beta^2+\beta^3)\ln(I_t\,y_t) + (\beta^2+\beta^3)(\ln(I_t\,y_t) + \ln(I_{t+1}\,y_{t+1})) + \beta^3(\ln(I_t\,y_t) + \ln(I_{t+1}\,y_{t+1}) + \ln(I_{t+2}\,y_{t+2}))\big)\\ &= \varphi\big(\beta(1+\beta+\beta^2)\ln(I_t\,y_t) + \beta^2(1+\beta)(\ln(I_t\,y_t) + \ln(I_{t+1}\,y_{t+1})) + \beta^3(\ln(I_t\,y_t) + \ln(I_{t+1}\,y_{t+1}) + \ln(I_{t+2}\,y_{t+2}))\big)\\ &= \varphi\left(\sum_{j=0}^{3}\beta^{j+1}\left(\sum_{n=1}^{j-1}\beta^n\right)\ln\left(I_{t+j}\,y_{t+j}\right)\right)\end{aligned}$$

This result shows that:

$$\begin{aligned}\lim_{h\to\infty}\sum_{s=t}^{h}\beta^{s-t}\left(\varphi\sum_{j=t}^{s-1}\ln\left(I_j y_j\right)\right) &= \lim_{h\to\infty}\varphi\left(\sum_{j=0}^{h}\beta^{j+1}\left(\sum_{n=1}^{j-1}\beta^n\right)\ln\left(I_{t+j}\,y_{t+j}\right)\right)\\ &= \lim_{h\to\infty}\varphi\left(\sum_{j=0}^{h}\beta^{j+1}\left(\frac{1}{1-\beta}\right)\ln\left(I_{t+j}\,y_{t+j}\right)\right)\\ &= \lim_{h\to\infty}\varphi\left(\sum_{j=0}^{h}\beta^{j}\left(\frac{\beta}{1-\beta}\right)\ln\left(I_{t+j}\,y_{t+j}\right)\right)\\ &= \varphi\left(\sum_{s=t}^{\infty}\beta^{s-t}\left(\frac{\beta}{1-\beta}\right)\ln(I_s\,y_s)\right)\end{aligned}$$

Taking these results, v_t can be rearranged as:

$$v_t = \sum_{s=t}^{\infty} \beta^{s-t} \left\{ (1+\omega)\ln(z_s) + u(k_s) + (1+\omega)\ln(y_s) + \omega \mathbb{E}\left[ln\left(G_{i,s}^{\chi} \left(\sum_{j=1}^{n} G_{j,s} \right)^{1-\chi} \right) \right] \right\}$$

$$= \frac{1+\omega}{1-\beta}\ln(z_t) + \sum_{s=t}^{\infty} \beta^{s-t} \left\{ (1+\omega)\left(\varphi \sum_{j=t}^{s-1} \ln\left(I_j y_j\right) \right) + u(k_s) + (1+\omega)\ln(y_s) + \omega \mathbb{E}\left[ln\left(G_{i,s}^{\chi} \left(\sum_{j=1}^{n} G_{j,s} \right)^{1-\chi} \right) \right] \right\}$$

$$= \frac{1+\omega}{1-\beta}\ln(z_t) + \sum_{s=t}^{\infty} \beta^{s-t} \left\{ \left(\frac{\beta\varphi(1+\omega)}{1-\beta} \right) \ln(I_s y_s) + u(k_s) + (1+\omega)\ln(y_s) + \omega \mathbb{E}\left[ln\left(G_{i,s}^{\chi} \left(\sum_{j=1}^{n} G_{j,s} \right)^{1-\chi} \right) \right] \right\}$$

Define *U (k, G, I)* as follows:

$$\begin{aligned} U(p_t) &= \left(\frac{\beta\varphi(1+\omega)}{1-\beta} \right) \ln(I_s\, y_s) + u(k_s) + (1+\omega)\ln(y_s) \\ &= \left(\frac{\beta\varphi(1+\omega)}{1-\beta} \right) \ln(I_s) + u(k_s) + (1+\omega)\left(\frac{\beta\varphi}{1-\beta} + 1 \right) \ln(y_s) \\ &= \left(\frac{\beta\varphi(1+\omega)}{1-\beta} \right) \ln(I_s) + u(k_s) + (1+\omega)\left(\frac{\beta(\varphi-1)+1}{1-\beta} \right) \ln(y_s) \end{aligned}$$

Now, define *V(k)* as

$$\mathcal{V}(k) = U(k, G(k), I(k)) + \omega \mathbb{E}\left[\ln\left(G_i^{\chi} \left(\sum_{j=1}^{n} G_j \right)^{1-\chi} \right) \right] + \beta \mathcal{V}(k'(k, G, I))$$

Expanding the expected value results in the following. Also, parametric terms can be eliminated since their derivative is equal to zero.

$$\begin{aligned} &\mathbb{E}\left[ln\left(G_i^{\chi} \left(\sum_{j=1}^{n} G_j \right)^{1-\chi} \right) \right] = \chi \mathbb{E}[\ln(G_i)] + (1-\chi)\mathbb{E}[\ln(G^T)] \\ &= \chi \left(\frac{1}{n-m+1} \ln(G) + \left(\frac{n-m}{n-m+1} \right) \ln\left(\underline{G}\right) \right) + (1-\chi)\ln(G^T) \\ &= \frac{\chi}{n-m+1} \ln(G) + \left(\frac{\chi(n-m)}{n-m+1} \right) \ln\left(\underline{G}\right) + (1-\chi)\ln(G^T) \end{aligned}$$

Therefore, *V(k)* can be arranged as:

$$\mathcal{V}(k) = U(k, G, I) + \omega \left(\frac{\chi}{n-m+1} \ln(G) + \left(\frac{\chi(n-m)}{n-m+1} \right) \ln\left(\underline{G}\right) + (1-\chi)\ln\left(G^T\right) \right) + \beta \mathcal{V}(k', G, I))$$

Since the therm $\omega\left(\left(\frac{\chi(n-m)}{n-m+1}\right)\ln(\underline{G})\right)$ is parametric, it can be removed in order to express the value function as:

$$\mathcal{V}(k) = U(k,G,I) + \frac{\omega\,\chi}{n-m+1}\ln(G) + \omega\,(1-\chi)\ln\,(G^T) + \beta\mathcal{V}(k',G,I)) \quad (15)$$

Since the proposer has the chance to select the level of public goods, the expected value for the present period disappears and *V (k, G, I)* can be rearranged as :

$$\mathcal{V}\,(k,G,I) = U(k,G,I) + \omega\chi\ln(G(k)) + \omega(1-\chi)\ln(G^T(k)) + \beta\mathcal{V}(k'(k,G,I))$$

Therefore, if $\mathcal{A} = \frac{1+\omega}{1-\beta}$ then $v_t = \mathcal{A}\ln(z_t) + \mathcal{V}(k,G,I)$ can be expressed with $\mathcal{A}$ as a constant and *V (k, G, I)* does not depend on productivity levels. This allows for the SME to play out.

The proposer's problem can be rearranged in the following fashion. First, the dynamic of productivity growth is substituted into the government budget. Second, the interest rate is substituted from the market determined interest rate. Third, total output is substituted into the value function, and lastly the indirect labour supply is substituted as well, since the government can anticipate the effect of its decisions on agents.

$$\mathcal{V}(k) = \max_{\mathcal{P}}\left\{\begin{array}{c}\left(\frac{\beta\varphi(1+\omega)}{1-\beta}\right)\ln(I) + u(C(k,G,I)) + (1+\omega)\left(\frac{\beta(\varphi-1)+1}{1-\beta}\right)\ln\,(y) \\ +\omega\chi\ln\big(G(k)\big) + \omega(1-\chi)\ln\,(G^T(k) + \beta\mathcal{V}(\mathcal{H}(k,G,I))\end{array}\right\}$$

$$\text{subject to}\left\{\begin{array}{c}\mathcal{T}(k,G,I) = \frac{y(I+G^T(k))}{\alpha y - \delta k} \\ G^T(k) = G(k) + (m-1)\left(G(k)^{\frac{1}{n-m+1}}\underline{G}^{1-\frac{1}{n-m+1}}\right) + (n-m)\underline{G} \\ y = k^\alpha L^{1-\alpha} \\ G \geq \underline{G}, I \geq \underline{I}, \tau \in [0,1]\end{array}\right\}$$

where:

$$\begin{cases}u(C) = \ln(C(k,G,I)) - \ln(n) \\ C(k,G,I) = 1 - \left(\frac{\mathcal{Z}}{y}\right)\mathcal{H} + (1-\delta)\frac{k}{y} - (I+G^T).\end{cases}$$

A4.b. Solution to the proposer's problem

For simplicity, define the following constants $A_0 \equiv \left(\frac{\beta\varphi(1+\omega)}{1-\beta}\right)$, $A_1 \equiv (1+\omega)\left(\frac{\beta(\varphi-1)+1}{1-\beta}\right)$, .Hence:

$$\mathcal{V}(k) = \max_{\mathcal{P}} \left\{ \begin{matrix} A_0 \ln(I) + u(C(k,G,I)) + A_1 \ln(y) \\ \\ +\omega\chi \ln(G(k)) + \omega(1-\chi)\ln(G^T(k)) + \beta\mathcal{V}(\mathcal{H}(k,G,I)) \end{matrix} \right\}$$

Because the tax rate can be solved for using the balanced budget restriction, the restrictions can be ignored after substituting for Y and G^T. The first order conditions for the proposer's problem be solved, with superscripts denoting forward lags and sub indices partial derivatives. Remember that consumption is:

$$C(k,G,I) = 1 - \left(\frac{z}{y}\right)\mathcal{H} + (1-\delta)\frac{k}{y} - (I + G^T)$$

Also, define for simplicity the function $F(G(k)) = \omega\chi\ln(G(k)) + \omega(1-\chi)\ln(G^T(k))$ which aggregates the direct utility effect of current government spending.

Therefore the FOC for *I* and *G* are:

Investment *I*:

$$\frac{A_0}{I} + \frac{\partial u(C)}{\partial I} + \beta\mathcal{V}_k'\mathcal{H}_I = 0$$

$$\frac{A_0}{I} + u_c\left(-1 - \left(\frac{z}{y}\mathcal{H}_I + \frac{z_I}{y}\mathcal{H}\right)\right) + \beta\mathcal{V}_k'\mathcal{H}_I = 0$$

Public good *G*:

$$\frac{\partial u(C)}{\partial G} + \frac{\partial F\big(G(k)\big)}{\partial G} + \beta\mathcal{V}_k'\mathcal{H}_G = 0$$

$$u_c\left(-\frac{\partial G^T}{\partial G} - \frac{Z}{y}\mathcal{H}_G\right) + F_G + \beta\mathcal{V}_k'\mathcal{H}_G = 0$$

$$\beta\mathcal{V}_k' = \frac{1}{\mathcal{H}_G}\left(u_c\left(\frac{\partial G^T}{\partial G} + \frac{Z}{y}\mathcal{H}_G\right) - F_G\right)$$

Where:

$$\frac{\partial G^T}{\partial G} = 1 + \left(\frac{m-1}{n-m+1}\right)\left(\frac{G}{G(k)}\right)^{1-\frac{1}{n-m+1}}$$

$$F_G = \frac{\omega\chi}{G(k)} + \frac{\omega(1-\chi)}{G^T(k)}\frac{\partial G^T}{\partial G}$$

Combining both FOC yields the **intra-temporal condition**: (Investment vs public goods)

$$\frac{A_0}{I}+\frac{\mathcal{H}_I}{\mathcal{H}_G}\left(u_c\left(\frac{\partial G^T}{\partial G}+\frac{Z}{y}\mathcal{H}_G\right)-F_G\right)=u_c\left(1+\left(\frac{Z}{y}\mathcal{H}_I+\frac{Z_I}{y}\mathcal{H}\right)\right)$$

$$\frac{A_0}{I}+u_c\left(\frac{\partial G^T}{\partial G}\frac{\mathcal{H}_I}{\mathcal{H}_G}+\frac{Z}{y}\mathcal{H}_I\right)-F_G\frac{\mathcal{H}_I}{\mathcal{H}_G}=u_c\left(1+\frac{Z}{y}\mathcal{H}_I+\frac{Z_I}{y}\mathcal{H}\right)$$

$$\frac{A_0}{I}-F_G\frac{\mathcal{H}_I}{\mathcal{H}_G}=u_c\left(1+\frac{Z_I}{y}\mathcal{H}-\frac{\partial G^T}{\partial G}\frac{\mathcal{H}_I}{\mathcal{H}_G}\right)$$

$$\frac{A_0}{I}=u_c\left(1+\frac{Z_I}{y}\mathcal{H}\right)+\frac{\mathcal{H}_I}{\mathcal{H}_G}\left(F_G-u_c\left(\frac{\partial G^T}{\partial G}\right)\right)$$

$$\frac{A_0}{I}=u_c\left(1+\frac{Z_I}{y}\mathcal{H}\right)+\frac{\mathcal{H}_I}{\mathcal{H}_G}\left(F_G-u_c\left(\frac{\partial G^T}{\partial G}\right)\right)$$

The derivation of the **inter-temporal condition**, the Generalized Euler Equation (GEE) is based on (Klein et al. 2008) method. First, the FOC are obtained. Second, the value functions is differentiated against *K*. Third, the FOC are applied to the V_k in order to eliminate various partial derivatives. Fourth, V_k is iterated one period forward and combined again with the FOC in order to obtain the GEE.

$$\mathcal{V}_k=\frac{A_0}{I}\,I_k+\frac{\alpha A_1}{k}+F_G G_k+\beta\mathcal{V}_k'(\mathcal{H}_k+\mathcal{H}_G G_k+\mathcal{H}_I I_k)+u_c\left(\frac{\partial C(k,G,I)}{\partial k}\right)$$

Since the households savings function is a reaction to *k, G, I*, then:

$$\frac{\partial\mathcal{H}(k,G(k),I(k))}{\partial k}=\mathcal{H}_k+\mathcal{H}_G G_k+\mathcal{H}_I I_k$$

and the derivative of consumption to becomes:

$$\frac{\partial C(k,G,I)}{\partial k}=-I_k-\frac{\partial G^T}{\partial G}\,G_k-\frac{\partial}{\partial k}\left[\left(\frac{Z}{y}\right)\mathcal{H}+(1-\delta)\frac{k}{y}\right]$$
$$=\;-I_k-\frac{\partial G^T}{\partial G}\,G_k-\frac{(1-\alpha)(1-\delta)}{y}-\frac{\partial}{\partial k}[I^{\varphi}y^{\varphi-1}\mathcal{H}]$$

Now, the product rule is applied to the last term:

$$\frac{\partial}{\partial k}[I^{\varphi}y^{\varphi-1}\mathcal{H}]=\left[(\varphi I^{\varphi-1}I_k)y^{\varphi-1}\mathcal{H}+I^{\varphi}\left((\varphi-1)y^{\varphi-2}\alpha\frac{y}{k}\right)\mathcal{H}+I^{\varphi}y^{\varphi-1}(\mathcal{H}_k+\mathcal{H}_G G_k+\mathcal{H}_I I_k)\right]$$
$$=\left[\varphi(Iy)^{\varphi-1}\mathcal{H}I_k+(\varphi-1)I^{\varphi}\left(y^{\varphi-1}\frac{\alpha}{k}\right)\mathcal{H}+\frac{(Iy)^{\varphi}}{y}(\mathcal{H}_k+\mathcal{H}_G G_k+\mathcal{H}_I I_k)\right]$$
$$=\left[\frac{Z_i}{y}\mathcal{H}I_k+(\varphi-1)\frac{Z}{y}\left(\frac{\alpha}{k}\right)\mathcal{H}+\frac{Z}{y}(\mathcal{H}_k+\mathcal{H}_G G_k+\mathcal{H}_I I_k)\right]$$
$$=\left[\frac{Z_i}{y}\mathcal{H}I_k+\frac{Z}{y}\left(\frac{(\varphi-1)\alpha\mathcal{H}}{k}+(\mathcal{H}_k+\mathcal{H}_G G_k+\mathcal{H}_I I_k)\right)\right]$$

Therefore, V_k can be expressed as:

$$\left\{\begin{array}{c} \mathcal{V}_k = \frac{A_0}{I} I_k + \frac{\alpha A_1}{k} + F_G G_k + \beta \mathcal{V}'_k \mathcal{H}_k + \beta \mathcal{V}'_k \mathcal{H}_G G_k ¿ \beta \mathcal{V}'_k \mathcal{H}_I I_k \\ +u_c \left(-I_k - \frac{\partial G^T}{\partial G} G_k - \frac{(1-\alpha)(1-\delta)}{y} - \frac{Z_I}{y} \mathcal{H} I_k - (\varphi - 1) \frac{Z}{y} \left(\frac{\alpha}{k}\right) \mathcal{H} - \frac{Z}{y} (\mathcal{H}_k + \mathcal{H}_G G_k + \mathcal{H}_I I_k) \right) \end{array}\right\}$$

This expression can be further reduced using the FOC for private investment and public goods by collecting the terms with I_k and G_k. Reorganizing:

$$\left\{\begin{array}{c} \mathcal{V}_k = \frac{A_0}{I} I_k + \beta \mathcal{V}'_k \mathcal{H}_I I_k + u_c \left(-I_k - \frac{Z_I}{y} \mathcal{H} I_k - \frac{Z}{y} \mathcal{H}_I I_k \right) \\ + F_G G_k + \beta \mathcal{V}'_k \mathcal{H}_G G_k + u_c \left(- \frac{\partial G^T}{\partial G} G_k - \frac{Z}{y} \mathcal{H}_G G_k \right) \\ + \frac{\alpha A_1}{k} + u_c \left(- \frac{(1-\alpha)(1-\delta)}{y} - \frac{Z}{y} \left((\varphi - 1) \left(\frac{\alpha}{k}\right) \mathcal{H} + \mathcal{H}_k \right) \right) + \beta \mathcal{V}'_k \mathcal{H}_k \end{array}\right\}$$

The first two lines of the expression are the FOC multiplied by I_k and G_k respectively. Therefore if the solution is optimal, the first two lines become zero. Hence V_k becomes:

$$\mathcal{V}_k = \frac{\alpha A_1}{k} + u_c \left(- \frac{(1-\alpha)(1-\delta)}{y} - \frac{Z}{y} \left((\varphi - 1) \left(\frac{\alpha}{k}\right) \mathcal{H} + \mathcal{H}_k \right) \right) + \beta \mathcal{V}'_k \mathcal{H}_k \quad (17)$$

In order to eliminate the term V'_k and derive the government's euler equation, express V'_k in terms of the FOC for G:

$$\beta \mathcal{V}'_k = \frac{1}{\mathcal{H}_G} \left(u_c \left(\frac{\partial G^T}{\partial G} + \frac{Z}{y} \mathcal{H}_G \right) - F_G \right)$$

Substituting into equation 17 gives:

$$\mathcal{V}_k = \frac{\alpha A_1}{k} + u_c \left(- \frac{(1-\alpha)(1-\delta)}{y} - \frac{Z}{y} \left((\varphi - 1) \left(\frac{\alpha}{k}\right) \mathcal{H} + \mathcal{H}_k \right) \right) + \frac{\mathcal{H}_k}{\mathcal{H}_G} \left(u_c \left(\frac{\partial G^T}{\partial G} + \frac{Z}{y} \mathcal{H}_G \right) - F_G \right)$$

Iterating one period forward:

$$\mathcal{V}'_k = \frac{\alpha A_1}{\mathcal{H}} + u_c \left(- \frac{(1-\alpha)(1-\delta)}{y'} - \frac{Z'}{y'} \left((\varphi - 1) \left(\frac{\alpha}{k}\right) \mathcal{H}' + \mathcal{H}'_k \right) \right) + \frac{\mathcal{H}'_k}{\mathcal{H}'_G} \left(u'_c \left(\frac{\partial G^{T\prime}}{\partial G'} + \frac{Z'}{y'} \mathcal{H}'_G \right) - F'_G \right)$$

Finally, this last result can be added to the combined FOC in order to obtain the Generalized Euler's Equation:

$$\left\{\begin{array}{c} u_c \left(1 - \frac{\partial G^T}{\partial G} - \frac{Z}{y} (\mathcal{H}_G - \mathcal{H}_I) - \frac{Z_I}{y} \mathcal{H} \right) + F_G - \frac{A_0}{I} = \\ \beta (\mathcal{H}_I - \mathcal{H}_G) \left[\frac{\alpha A_1}{\mathcal{H}} + u_c \left(- \frac{(1-\alpha)(1-\delta)}{y'} - \frac{Z'}{y'} \left((\varphi - 1) \left(\frac{\alpha}{\mathcal{H}}\right) \mathcal{H}' + \mathcal{H}'_k \right) \right) + \frac{\mathcal{H}'_k}{\mathcal{H}'_G} \left(u'_c \left(\frac{\partial G^{T\prime}}{\partial G'} + \frac{Z'}{y'} \mathcal{H}'_G \right) - F'_G \right) \right] \end{array}\right\}$$

Summary of the government's optimality conditions (primes denote forward lags, subscripts partial derivatives):

Intra-temporal condition (*I* vs *G*):

$$\frac{A_0}{I} = u_c\left(1 + \frac{Z_I}{y}\mathcal{H}\right) + \frac{\mathcal{H}_I}{\mathcal{H}_G}\left(F_G - u_c\left(\frac{\partial G^T}{\partial G}\right)\right)$$

Inter-temporal condition (GEE):

$$\left\{\begin{array}{c} u_c\left(1 - \frac{\partial G^T}{\partial G} - \frac{Z}{y}(\mathcal{H}_G - \mathcal{H}_I) - \frac{Z_I}{y}\mathcal{H}\right) + F_G - \frac{A_0}{I} = \\ \beta(\mathcal{H}_i - \mathcal{H}_G)\left[\frac{\alpha A_1}{\mathcal{H}} + u_c\left(-\frac{(1-\alpha)(1-\delta)}{y'} - \frac{Z'}{y'}\left((\varphi-1)\left(\frac{\alpha}{\mathcal{H}}\right)\mathcal{H}' + \mathcal{H}'_k\right)\right) + \frac{\mathcal{H}'_k}{\mathcal{H}'_G}\left(u'_c\left(\frac{\partial G^T}{\partial G'} + \frac{Z'}{y'}\mathcal{H}'_G\right) - F'_G\right)\right] \end{array}\right\}$$

Tax-rate $\tau = T(k,G(k),I(k))$:

$$\mathrm{T}\big(k, G(k), I(k)\big) = y\left(\frac{I(k) + G^T(k)}{\alpha y - \delta k}\right)$$

A4.c. Simultaneous game

When the government optimises taking $k' = H(k, G(k), I(k))$ as given (simultaneous game).

its optimality conditions become:

Demand for *I*: $\frac{A_0}{I} = u_c\left(1 + \frac{Z_i}{y}\mathcal{H}\right)$

Demand for *G*: $F_G = \frac{A_0}{I} - u_c\left(1 - \frac{\partial G^T}{\partial G} + \frac{Z_I}{y}\mathcal{H}\right)$

Tax-rate $\tau = T$: $\mathrm{T}\big(k, G(k), I(k)\big) = y\left(\frac{I(k)+G^T(k)}{\alpha y - \delta k}\right)$

Lastly, the budget constraint implies that the government will set the tax rate in order to balance its budget. However, both and will depend on the other two optimality conditions which in turn depend on how the government anticipates households to react to its fiscal policy platform. Therefore, an interior solution is possible since the government knows that if it collects all capital gains as taxes, then households will consume less in the future. Hence a marginal reduction in tax rates today increase disposable income, thus affecting the household's consumption and savings reaction function.

Rearranging the GEE:

$$\left\{\begin{array}{c} (\mathcal{H}_I - \mathcal{H}_G)\left(u_c\frac{Z}{y} - \beta u'_c\frac{Z'}{y'}\mathcal{H}'_k\right) + u_c\left(1 - \frac{\partial G^T}{\partial G} - \frac{Z'}{y}\mathcal{H}\right) + F_G - \frac{A_0}{I} = \\ \beta(\mathcal{H}_I - \mathcal{H}_G)\left[\frac{\alpha A_1}{\mathcal{H}} + u_c\left(-\frac{(1-\alpha)(1-\delta)}{y'} - \frac{Z'}{y'}\left(\alpha(\varphi-1)\left(\frac{\mathcal{H}'}{\mathcal{H}}\right) + \mathcal{H}'_k\right)\right) + \frac{\mathcal{H}'_k}{\mathcal{H}'_G}\left(u'_c\left(\frac{\partial G^{T'}}{\partial G'}\right) - F'_G\right)\right] \end{array}\right\}$$

The household's euler equation is rearranged with *nc = zyC (k, G, I)*

$$\frac{c'}{c} = \beta(1 + (1-\tau')r')$$
$$\frac{z'y'C'}{zyC} = \beta(1 + (1-\tau')r')$$
$$\mathcal{Z}\frac{y'C'}{yC} = \beta(1 + (1-\tau')r')$$
$$\frac{\mathcal{Z}}{y}u_c = \beta\frac{u_c'}{y'}(1 + (1-\tau')r')$$

Hence the GEE can be combined with the household's euler:

$$\left\{\begin{array}{c} \beta(\mathcal{H}_I - \mathcal{H}_G)\left[\frac{u_c'}{y'}\left((1 + (1-\tau')r') - Z'\mathcal{H}_k'\right)\right] + u_c\left(1 - \frac{\partial G^T}{\partial G} - \frac{Z_I}{y}\mathcal{H}\right) + F_G - \frac{A_0}{I} = \\ \beta(\mathcal{H}_I - \mathcal{H}_G)\left[\frac{\alpha A_1}{\mathcal{H}} + u_c\left(-\frac{(1-\alpha)(1-\delta)}{y'} - \frac{Z'}{y'}\left(\alpha(\varphi-1)\left(\frac{\mathcal{H}'}{\mathcal{H}}\right) + \mathcal{H}_k'\right)\right) + \frac{\mathcal{H}_k'}{\mathcal{H}_G'}\left(u_c'\left(\frac{\partial G^{T\prime}}{\partial G'}\right) - F_G'\right)\right] \end{array}\right\}$$

A4.d. Simultaneous game

Suppose that the game is simultaneous. Therefore, the government does not internalize the household's reaction functions and proposes its policy with *H* taken as given. Therefore, from the government perspective it cannot anticipate the effects on consumers of its policies. Hence, $H_I = H_G = 0$.

Therefore the FOC for and are:

Investment *I:* $\frac{A_0}{I} + u_c\left(-1 - \frac{Z_I}{y}\mathcal{H}\right) = 0$

Public good *G:* $u_c\left(-\frac{\partial G^T}{\partial G}\right) + F_G = 0$

Where:

$$\frac{\partial G^T}{\partial G} = 1 + \left(\frac{m-1}{n-m+1}\right)\left(\frac{G}{G(k)}\right)^{1-\frac{1}{n-m+1}}$$
$$F_G = \frac{\omega\chi}{G(k)} + \frac{\omega(1-\chi)}{G^T(k)}\frac{\partial G^T}{\partial G}$$

Combining both FOC yields the **intra-temporal condition**: (Investment vs public goods) and thus the government's optimality conditions become:

Demand for *I:* $\frac{A_0}{I} = u_c\left(1 + \frac{Z_I}{y}\mathcal{H}\right)$

I vs *G* mix: $u_c\left(1 - \frac{\partial G^T}{\partial G} + \frac{Z_I}{y}\mathcal{H}\right) + F_G - \frac{A_0}{I} = 0$

Tax-rate $\tau = T(k,G(k), I(k))$: $\mathrm{T}\big(k, G(k), I(k)\big) = y\left(\frac{I(k) + G^T(k)}{\alpha y - \delta k}\right)$

Where:

$$\mathcal{H}(k,G,I)=k'$$
$$u_c=\frac{y}{n\hat{c}}$$
$$\frac{Z_I}{y}=\varphi(Iy)^{\varphi-1}$$
$$\frac{\partial G^T}{\partial G}=1+\left(\frac{m-1}{n-m+1}\right)\left(\frac{\underline{G}}{G(b,k)}\right)^{1-\frac{1}{n-m+1}}$$

$$F_G=\frac{\omega\chi}{G(k)}+\frac{\omega(1-\chi)}{G^T(k)}\frac{\partial G^T}{\partial G}$$

A5. Welfare analysis

This is proposition 3 of the paper. Lifetime utility for a citizen can be expressed as given a productivity level z_s .

$$W=\left\{\begin{array}{c} n\left(\frac{1}{1-\beta}\right)\left(\ln(\hat{c}_s)+\omega\ln(y_s)+\ln(z_0)+\varphi\left(\frac{\beta}{1-\beta}\right)\ln(I_s y_s)\right)+ \\ \left(\frac{1}{1-\beta}\right)\left(\omega\chi\left(\ln(G)+(m-1)\ln(G_m)+(n-m)\ln(\underline{G})\right)+n(1-\chi)\ln(G^T)\right)\end{array}\right\} \quad (18)$$

Proof. Let the household's per-period utility function be:

$$\begin{aligned}\mathcal{U}_i(k_t,z_t)&=\ln(c_t)+\omega\ln\left(\mathcal{G}_{i,t}^{\chi}\left(\sum_{j=1}^{n}\mathcal{G}_{j,t}\right)^{1-\chi}\right)\\&=\ln(z_t\hat{c}_t)+\omega\ln\left(Y_t G_{i,t}^{\chi}\left(\sum_{j=1}^{n}G_{j,t}\right)^{1-\chi}\right)\\&=(1+\omega)\ln(z_t)+\ln(\hat{c}_t)+\omega\ln(y_t)+\omega\ln\left(G_{i,t}^{\chi}(G_t^T)^{1-\chi}\right)\end{aligned}$$

Hence expected lifetime utility can be arranged as:

$$\mathbb{E}\left[\sum_{t=0}^{\infty}\beta^t\,\mathcal{U}_i(k_t,z_t)\right]=\mathbb{E}\left[\sum_{t=0}^{\infty}\beta^t\left(\ln(\hat{c}_t)+\omega\ln(y_t)+\omega\ln\left(G_{i,t}^{\chi}(G_t^T)^{1-\chi}\right)\right)+\sum_{t=0}^{\infty}\beta^t\left((1+\omega)\ln(z_t)\right)\right]$$

In a balanced growth path (Definition 2), productivity-adjusted output y_s and productivity-adjusted consumption $\hat{c}_s$ are constant. Similarly, pork barrel as a share of total output remains constant and thus total public spending becomes constant. Therefore, the first term of the lifetime utility can be rearranged as:

$$\mathbb{E}\left[\sum_{t=0}^{\infty}\beta^t(\ln(\hat{c}_t)+\omega\ln(y_t)+\omega\ln\left(G_{i,t}^{\chi}(G_t^T)^{1-\chi}\right))\right]=\left(\frac{1}{1-\beta}\right)(\ln(\hat{c}_s)+\omega\ln(y_s))+\mathbb{E}\left[\sum_{t=0}^{\infty}\beta^t\omega\,ln\left(G_{i,t}^{\chi}(G_t^T)^{1-\chi}\right)\right]$$

The last term can be further rearranged as:

$$\mathbb{E}\left[\sum_{t=0}^{\infty}\beta^t\,\omega\ln\left(G_{i,t}^{\chi}(G_t^T)^{1-\chi}\right)\right]=\sum_{t=0}^{\infty}\beta^t\,\omega\mathbb{E}\left[\ln\left(G_{i,t}^{\chi}(G_t^T)^{1-\chi}\right)\right]$$
$$=\sum_{t=0}^{\infty}\beta^t\,\omega\mathbb{E}\left[\chi\ln\left(G_{i,t}\right)\right]+\sum_{t=0}^{\infty}\beta^t\,\omega(1-\chi)\ln(G_t^T)$$

Expanding the expected value results in the following.

$$\mathbb{E}\left[\ln\left(G_i^{\chi}\left(\sum_{j=1}^{n}G_j\right)^{1-\chi}\right)\right]=\chi\mathbb{E}[\ln(G_i)]+(1-\chi)\mathbb{E}[\ln(G^T)]$$
$$=\frac{\chi}{n}\left(\ln(G)+(m-1)\ln(G_m)+(n-m)\ln\left(\underline{G}\right)\right)+(1-\chi)\ln(G^T)$$

Therefore

$$\mathbb{E}\left[\sum_{t=0}^{\infty}\beta^t\omega\,ln\left(G_{i,t}^{\chi}(G_t^T)^{1-\chi}\right)\right]=\sum_{t=0}^{\infty}\beta^t\omega\left(\frac{\chi}{n}\left(\ln(G)+(m-1)\ln(G_m)+(n-m)\ln\left(\underline{G}\right)\right)+(1-\chi)\ln\left(G^T\right)\right)$$
$$=\left(\frac{1}{1-\beta}\right)\left(\frac{\omega\chi}{n}\left(\ln(G)+(m-1)\ln(G_m)+(n-m)\ln\left(\underline{G}\right)\right)+(1-\chi)\ln\left(G^T\right)\right)$$

Therefore, in a balanced growth path the following expression is a constant and depends on the steady-state levels of $\hat{c_s}$, y_s, G_s.

$$\mathbb{E}\left[\sum_{t=0}^{\infty}\beta^t\mathcal{U}_i(k_t,z_t)-(1+\omega)\ln\left(z_t\right)\right]$$
$$=\left\{\begin{array}{c}\left(\frac{1}{1-\beta}\right)\left(\ln(\hat{c_s})+\omega\ln(y_s)\right)+\\ \left(\frac{1}{1-\beta}\right)\left(\frac{\omega\chi}{n}\left(\ln(G)+(m-1)\ln(G_m)+(n-m)\ln\left(\underline{G}\right)\right)+(1-\chi)\ln\left(G^T\right)\right)\end{array}\right\}$$

Lastly, the expression for the present value of productivity growth becomes:

$$\mathbb{E}\left[\sum_{t=0}^{\infty}\beta^t\,(1+\omega)\ln(z_t)\right]=$$

Using the the expression for productivity growth:

$$\ln(z_{t+1})=\varphi\ln(I_s y_s)+\ln(z_t)$$
$$\ln(z_{t+2})=\varphi\ln(I_s y_s)+\ln(z_{t+1})$$
$$\vdots$$
$$\ln(z_{t+h})=h\,\varphi\ln(I_s y_s)+\ln(z_t)$$

Therefore:

$$\sum_{t=0}^{\infty} \beta^t \ln(z_t) = \ln(z_0) + \beta \ln(z_1) + \beta^2 \ln(z_2) + \beta^3 \ln(z_3) + \cdots$$
$$= (1 + \beta + \beta^2 + \beta^3 + \cdots) \ln(z_0) + \beta\varphi(\ln(I_s y_s) + 2\beta \ln(I_s y_s) + 3\beta^3 (I_s y_s) + \cdots)$$
$$= \left(\frac{1}{1-\beta}\right) \ln(z_0) + \beta\varphi \sum_{t=0}^{\infty} t\beta^t \ln(I_s y_s)$$
$$= \left(\frac{1}{1-\beta}\right) \ln(z_0) + \beta\varphi \left(\frac{1}{(1-\beta)^2}\right) \ln(I_s y_s)$$
$$= \left(\frac{1}{1-\beta}\right) \left(\ln(z_o) + \varphi \left(\frac{\beta}{1-\beta}\right) \ln(I_s y_s)\right)$$

Therefore, aggregate wellfare *W* given an initial productivity level starting off at a balanced growth path becomes:

$$W = \left\{ \begin{array}{c} n\left(\frac{1}{1-\beta}\right)\left(\ln(\hat{c}_s) + \omega \ln(y_s) + \ln(z_0) + \varphi \left(\frac{\beta}{1-\beta}\right) \ln(I_s y_s)\right) + \\ \left(\frac{1}{1-\beta}\right)\left(\omega\chi\left(\ln(G) + (m-1)\ln(G_m) + (n-m)\ln(\underline{G})\right) + n(1-\chi)\ln(G^T)\right) \end{array} \right\}$$